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ABSTRACT

A large-scale, summative, comparative evaluation of "Patterns in Arithmetic (PIA)", a modern televised arithmetic curriculum for grades one through six, was carried out in grades one through four in both rural and urban schools during the 1970-71 school year. This report deals with the ways in which PIA affects teachers: in basic mathematical knowledge, knowledge of PIA-specific content, and attitudes toward teaching arithmetic. Findings indicate that PIA can be used effectively as in-service education, particularly for those teachers with relatively lower initial knowledge of the basic mathematics which underlies a contemporary elementary school mathematics program. PIA does not seem to change teachers' attitudes, however; nor is it beneficial in increasing knowledge of concepts not specifically related to PIA. These results seem to hold equally for both rural and urban schools. (Author/JM)

EVALUATION OF PATTERNS IN ARITHMETIC

IN GRADES 1-4, 1970-71: EFFECTS ON TEACHERS

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EVALUATION OF PATTERNS IN ARITHMETIC IN GRADES 1-4, 1970-71: EFFECTS ON TEACHERS

by J. Laird Marshall and Thomas J. Fischbach

Report from the
Quality Verification Component of Program 5
Mary R. Quilling, Program Coordinator

Wisconsin Research and Development
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Statement of Focus

The Wisconsin Research and Development Center for Cognitive Learning focuses on contributing to a better understanding of cognitive learning by children and youth and to the improvement of related educational practices. The strategy for research and development is comprehensive. It includes basic research to generate new knowledge about the conditions and processes of learning and about the processes of instruction, and the subsequent development of research-based instructional materials, many of which are designed for use by teachers and others for use by students. These materials are tested and refined in school settings. Throughout these operations behavioral scientists, curriculum experts, academic scholars, and school people interact, insuring that the results of Center activities are based soundly on knowledge of subject matter and cognitive learning and that they are applied to the improvement of educational practice.

This Technical Report is from the Quality Verification Program, whose principal function is to identify and invent research and development strategies taking into account current knowledge in the field of statistics, psychometrics and computer technology. The Quality Verification Program collaborates in applying such strategies in research and development. The translation of theory into practice and presentations of exemplars of methodology are challenges which the Quality Verification Program strives to meet.

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Abstract

A large-scale, summative, comparative evaluation of Patterns in Arithmetic, a modern televised arithmetic curriculum for Grades 1-6, was carried out in Grades 1-4 in both rural and urban schools during the 1970-71 school year. This report deals with the ways in which PIA affects teachers: in basic mathematical knowledge, knowledge of PIA-specific content, and attitudes toward teaching arithmetic. Findings indicate that PIA can be used effectively as in-service education, particularly for those teachers with relatively lower initial knowledge of the basic mathematics which underlies a contemporary elementary school mathematics program. PIA does not seem to change teachers' attitudes, however; nor is it beneficial in increasing knowledge of concepts not specifically related to PIA.

These results seem to hold equally for both rural and urban schools.

I
Introduction

Background

Patterns in Arithmetic (PIA), a comprehensive elementary mathematics program for Grades 1-6, originated in 1959 and was developed over the next decade under the leadership of Professor Henry Van Engen with the support of the Ford Foundation and, starting in 1964, the Wisconsin Research and Development Center for Cognitive Learning. The components of PIA are television lessons on videotape, teacher suggestion manuals, and pupil exercise books. When PIA is used as an instructional program, a class watches a 15-minute videotaped lesson once or twice a week (the schedule varies at different grade levels) and the teacher supplements this by using the PIA exercise books and suggestions in the PIA manual. The normal sequence of events would be: a few minutes of pre-telecast warm-up, suggestions for which come from the teacher's manual; the telecast, during which the pupils may be asked by the TV teacher to manipulate various objects or work a few problems; post-telecast activities, suggested in the manual, which enlarge upon or review the important concept presented in the telecast; and then, over the next few days, supplementary activities from the manual and exercises in the pupil exercise books, some of which are related to material in the telecast and some of which are review of concepts presented in previous telecasts. Short tests are also given a few times a semester so that teachers can assess pupil progress.

PIA was designed to be utilized in the period of transition to a modern mathematics program at the elementary level. To quote Professor Van Engen,

> Remember that the telecasts are not intended to replace the classroom teacher. They are intended to help introduce and demonstrate new mathematical ideas. You, the teacher, are still the most important element in the development of a sound and meaningful mathematics program in your school....PIA is one answer to a new program in mathematics, but after having used PIA for a year or two, you will be ready to find other answers for your school. Whatever happens, remember that change is inevitable. You must help determine the direction of that change.[1]

The dual objectives of PIA are to provide a sound program in mathematics for the elementary school child and to provide in-service education for the elementary school teacher. Television was chosen as the medium by which to accomplish these aims, particularly for pupils and teachers in rural areas, where distance and sparse population often make it difficult to carry out in-service meetings or visits, and in the centers of large cities, where high turnover in both pupil and teacher populations and other familiar ills frequently make educational stability and quality difficult to achieve.

For more complete information on the history, mathematical content, behavioral objectives, and pedagogical principles of PIA, see Braswell and Romberg (1969). For an overview of the key mathematical ideas in PIA, see Appendix A.

Previous Evaluations

As part of the research and development effort, a summative, noncomparative evaluation of the PIA program in Grades 1 and 3 was carried out during the 1966-67 school year (Braswell & Romberg, 1969). Several hundred first-

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[1] From the preface to the Patterns in Arithmetic Teacher's Manuals.

and third-grade classrooms in Alabama and Wisconsin took part in the study, carried out in communities of four different sizes, the two largest of which were further broken down into three socioeconomic levels. Very briefly, the results of that study indicated that most of the objectives set forth by the PIA staff were being accomplished. In addition, further analysis revealed no relation between performance and community size or socioeconomic level. The only difference appeared to be between the two states, and then only on the standardized computation test.

A field test for Grade 2 was carried out in the 1967-68 school year (Braswell, 1969a). Again the results indicated that PIA was accomplishing most of its goals. At the end of the school year, the children in the 30 classes in the study scored significantly higher than the norms on a standardized test of concepts for that grade level, despite having scored lower than the norms group at the beginning of the year.

A formative evaluation of the PIA program for Grade 5, with about 80 classes participating, was also carried out in 1967-68 (Braswell, 1969b). Results indicated that children learned traditional computation skills and important concepts in arithmetic commensurate with their stage of development. Computation problems tended to be easier at the end of the school year than they were shortly after the skills were presented, which is good evidence that skills learned early in the year are not forgotten, but are reinforced by the structure of the PIA pupil exercise books.

In the 1968-69 school year, the PIA staff at the Wisconsin Research and Development Center for Cognitive Learning conducted a formative evaluation of the PIA program for Grade 6 (Braswell, 1970). The study successfully used the techniques of item sampling to evaluate the year's curriculum. This evaluation engendered several specific improvements in the telecasts.

Reasons for Present Study

One of the original objectives in developing PIA was to provide in-service education for elementary school teachers, but none of the evaluations mentioned heretofore had investigated the effectiveness of PIA in this respect. Moreover, no evaluation of Grades 1-3 had been carried out following revision of the teacher's manuals for these three grades. The original manuals had been judged by teachers to be of limited use, and the revised versions included, for each telecast: a statement of behavioral objectives, a description of the mathematical background for the topics to be covered, an overview of the program itself, and specific descriptions of what to do prior to the telecast; the materials to be used by students during the telecast, what goes on in the television program itself, and what to do after the telecast is completed.

In addition, as a result of feedback regarding utilization of PIA in various settings, a need was felt to ascertain the effectiveness of PIA among rural and urban children, and again none of these studies had focused on this particular question. Thus a large-scale, summative, comparative evaluation was carried out in the 1970-71 school year, focusing both on teachers and on rural and urban pupils in Grades 1-4. (The study would have been extended longitudinally to Grades 5 and 6 were it not for limited funds.)

The attribute of a rural setting which makes televised education promising is the fact that many teachers have not had recent opportunity to update their skills. Thus, used properly—with the teachers viewing the program as their pupils see it—PIA becomes an in-service program in the concepts and pedagogy of modern mathematics. Both teachers and pupils learn the new ideas through the medium of television from qualified teachers presenting contemporary mathematics content.

Urban settings are often characterized by high transiency of pupils, and even teachers, within the school district. A televised program, in addition to being a familiar and appealing medium, provides for continuity in the basic skills program and is an educational touchstone for the transient pupils. Moreover, since urban children are frequently deficient in the reading skills required for efficient learning from mathematics textbooks, television programs in which manipulation of concrete materials is a key feature may be a particularly effective means of presenting new concepts.

There are some factors, of course, which are important in both types of demographic situations—television's appeal to children and its efficiency and economy as an in-service course.

Goals of the Study

In general, the data can be evaluated from three approaches: the effects of PIA on pupils, on teachers, and the interaction between the two. This report will concern itself only with the second kind: in what ways, if any, are

teachers affected by using PIA in their classrooms? Other reports in preparation will deal with the other two aspects.

The objectives to be dealt with in this report are:

1. Do teachers who use PIA increase their knowledge of modern mathematics; i.e., is PIA an effective in-service course?

2. Do teachers who use PIA become more positively disposed toward teaching mathematics, i.e., enjoy it more or become more confident?

3. Does the effect of PIA on teachers' knowledge vary with the

   a. demographic characteristics of the school?

   b. grade level taught?

   c. level of teachers' initial general knowledge of the mathematics which underlies contemporary arithmetic?

4. Does the effect of PIA on teachers' attitudes vary with these same three factors?

II
The Evaluation Plan

Subjects

The experiment was designed to be carried out in urban and rural areas. Urban districts selected were New York archdiocese schools, Chicago public schools, and the Portland, Oregon, school system. Rural sites were located in the state of Vermont and in three rural counties near Roanoke, Virginia. Within each of the five sites, schools willing to participate were identified and were randomly assigned to the experimental or the control group. In the experimental schools, PIA was to be used as the major or sole component in the mathematics curriculum.

Approximately 10,000 children and 400 teachers in about 85 schools took part in the study. Because of various unforeseen reasons, some of which are explained later, this sample was reduced somewhat in the data analysis, but the numbers in the analysis are still of the same order of magnitude. Table 1 indicates which grades participated in which sites, and gives approximate numbers of children, teachers and schools in each site. It is sufficient to note that within each site there were approximately equal numbers of experimental and control schools. In those sites where more than one grade level participated, there were approximately equal numbers of classes in each grade.

Method

Classes in the experimental schools used the Patterns in Arithmetic programs as their sole or major component in the mathematics curriculum. This included the television programs, pupil exercise books, and the teacher's manuals, all described earlier.

Classrooms in the control schools proceeded normally through the school year with whatever mathematics curriculum they chose to use. Although teachers in control school classrooms were encouraged not to use the PIA telecasts, they were not prevented from doing so if they chose; in fact, some of the teachers did use the PIA telecasts, but generally as a supplement to another curriculum.

Measuring Instruments

At the beginning of the 1970-71 school year, teachers from both experimental and control schools attended an informational meeting. At this time, data were gathered by means of a questionnaire and a test of elementary mathematics. Near the end of the year, the participants were asked to fill out a second questionnaire and take a posttest.

The preliminary questionnaire, which follows as Appendix B, asked for biographical data, length and kind of teaching experience, and information about the recency and extent of training in mathematics and modern arithmetic concepts. Also included were questions about how much the teacher enjoyed teaching arithmetic compared with other subjects, and how confident the teacher felt when teaching the more difficult concepts of arithmetic. (The term "more difficult" was not defined, but rather left to the individual teacher's subjective judgment.)

The post-questionnaire, Appendix C, asked for information about the extent to which the PIA telecasts and supplementary materials were used and for teachers' opinions as to the appropriateness and effectiveness of the PIA package for their pupils, and their desire to use the program again next year. Two questions from the pre-questionnaire about the teacher's enjoyment of and confidence in teaching arithmetic were also included, so that attitudinal changes could be measured.

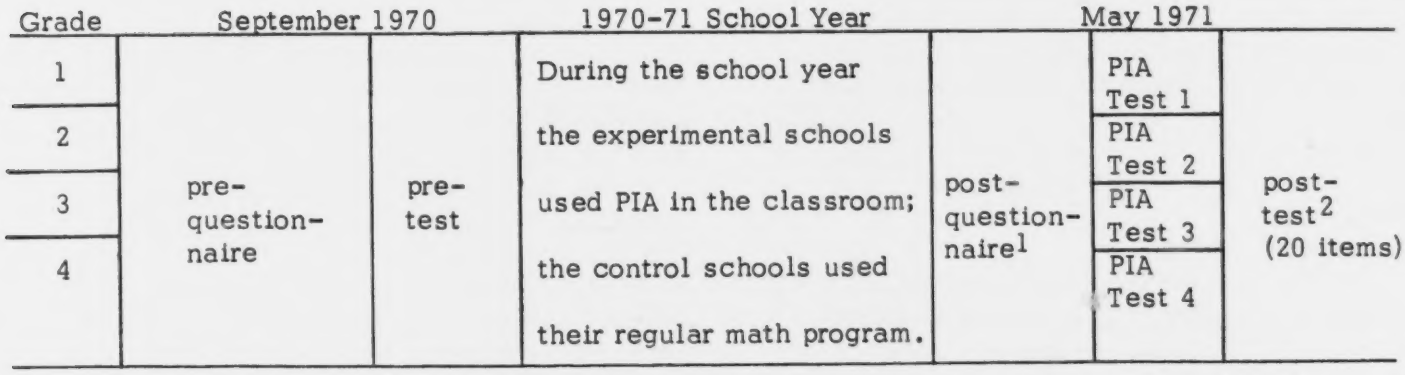
Table 1
Approximate Number of Participants in the Study

Site	Grade Level 1	2	3	4	Schools	Teachers	Pupils
New York			X	X	40	150	4700
Chicago	X	X			10	90	2350
Portland*	X	X			10	40	850
Vermont	X	X	X		10	70	1650
Virginia				X	15	40	1050
	TOTAL				85	390	10600

*Data not used in analysis.

The 50-item pretest, Appendix D, measured how familiar the teacher was with the basic mathematical concepts which underlie a contemporary elementary mathematics program. There were items on sets, number, numerals, numeration systems, operations, properties, informal geometry, and a few other topics or combinations of these topics. The intent of the test was not to measure those concepts or vocabulary in the PIA telecasts, but rather to get a broad measure of concepts basic to any modern approach to mathematics. The items were chosen from two sources: the SMSG Film—Film Text Evaluation Study Mathematics Inventory (School Mathematics Study Group, 1963) and the Sets and Systems Achievement Tests I-V (Educational Testing Service, 1964).

The test taken in May, 1971 (Appendix E) consisted of two parts: 15 items specially constructed to measure the specific content from the year's PIA telecasts; and a 20-item posttest, culled from the pretest on the bases of item analysis statistics and PIA/grade level content appropriateness. Although the 15-item PIA tests were constructed to measure only PIA content, care was taken to include only those concepts or vocabulary which control (non-PIA) teachers could reasonably be expected to have encountered in any general exposure to the basic concepts underlying modern arithmetic. These 15 items (PIA Tests 1, 2, 3, and 4) were necessarily different for the most part for the four grade levels, 1-4, in the experiment. However, the 20 items culled from the pretest were the same for all four grade levels. A diagram of the design follows:



[1]Two attitudinal questions in the pre-questionnaire were repeated in the post-questionnaire.

[2]Posttest items were imbedded in the pretest.

Fig. 1. Diagram of the experimental design.

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III
Analysis of the Data

Theory

We defined target population to be all teachers in the sites from which the data were sampled, as well as in sites to which the results of the study could reasonably be generalized. As such, the ultimate questions of research interest were stated in terms of two null test hypotheses:

1. If teachers in the target population use PIA, there will be no effect on their knowledge either of PIA content or of the basic general mathematical concepts and procedures (henceforth called "basic mathematics") which underlie contemporary mathematics for the elementary school.

2. If teachers in the target population use PIA, there will be no effect on either confidence in or enjoyment of teaching elementary school mathematics.

To determine the plausibility of the two hypotheses, two sources of data were available. One was a randomly selected sample from the target population who had not used PIA (the control group) but whose schools were willing to participate in the evaluation. The sample statistics from this source provided estimates of knowledge and attitudes existing in the target population at the end of the study. A second sample was similarly selected from the same original target population, but the teachers therein were exposed to PIA during the school year (the experimental group). The sample statistics from this source provided estimates of knowledge and attitudes at the end of the study that would have obtained had all teachers in the target population used PIA in the classroom.

If PIA had no effect, then these two data samples come from the same theoretical population and thus would yield the same performance levels and attitudes within the limits of sampling and measurement errors. Because of the random assignment to experimental or control groups of schools within each site and because measurement and sampling error distributions can be well-approximated by fairly simple statistical models, it was thus possible to compute statistical tests of the hypotheses listed above.

Preliminary Steps in the Analysis

The school was the unit of analysis since schools within a site were randomly assigned to either the experimental or control group. An advantage of using the mean for a school as the statistic of analysis, besides its convenient statistical properties, was the fact that if gain score means were used we would not need to be concerned about teacher turnover during the study: we could obtain unbiased estimates of unit (school) performance and unit gains regardless of whether the personnel changed (in some schools the personnel turnover was 50% in the grades studied).

Grade level was one type of grouping of teachers within units, and is a within-unit source of variation. A second type of grouping, within the sample as a whole, was site, another source of variance. Performance levels and/or attitudes might be expected to vary with both of these factors. This would be of no consequence to subsequent analysis if neither factor altered the effect of PIA on teachers' behavior, as measured with the present instruments, but if such interactions existed, it was considered to be important to discover and report them. Thus the first two steps in the analysis were done to determine whether the effect of

PIA on teachers varies by grade level or site. If the findings were negative, the statistical model for further analysis could be greatly simplified.

Step 1: Grade

Not all four grade levels were represented within any one site—the number was either one, two, or three (see Table 1, p. 6). However, sampling was done randomly within each site and resulted in essentially equal numbers of schools (and therefore all classes within a school) assigned to the experimental and control treatments by grade. Thus, grade and treatment were well balanced within each site.

If, within a site, PIA's effect did not seem to vary with grade level, grade could be ignored by averaging the grade means within each school. This would result in but one statistic (for each variable) for each school, and would simplify procedures still further and make hypothesis testing more powerful.

Multivariate repeated measures analyses of variance (with grade as the repeated measure) were used to test the hypotheses that the Grade x Treatment interaction effects are zero for each variable (Table 2). The dependent variables here, as elsewhere, deal with knowledge of mathematics and attitudes toward teaching it. The results, a multivariate significance level of 0.68, showed the hypothesis concerning knowledge is not unreasonable; in fact, the evidence against the hypothesis was not even of moderate strength. The same results (significance level of 0.76—see Table 3, p. 10) obtained with the hypothesis using attitude as the variate.[2]

Thus there was no reason to reject the hypothesis that Grade x Treatment interaction is zero for either the knowledge or the attitude variable; we can safely continue the analysis on the assumption that the effects of PIA on teachers' knowledge and attitudes does not vary with grade, and can proceed with the above-mentioned simplifications in the analysis model.

Step 2: Site

We now proceed to determine whether the effect of PIA varies according to site.[3] If it can be assumed that the Site x Treatment interaction effect is zero, then the analysis model can again be simplified and certain inferences may be made with greater precision. For each site, a weighted mean of the estimates of treatment effect (school means), where the weights are inversely proportional to the variances of the estimates, was used as the statistic in the analysis; this statistic, under the assumption of zero Site x Treatment interaction, has the smallest variance of all unbiased linear estimators, and therefore enhances precision. (For example, the mean from New York, as the site with the most cases [schools], would be given most weight.)

Since the assumption of zero interaction is the key to the final statistical model, the reasonableness of this hypothesis must be investigated. This was done in two ways: through the classical analysis of variance approach involving a test of this hypothesis, and by examination of "residuals," i.e., the differences between the values that would obtain if the model holds true and the actual values observed in the data samples, for each combination of site and treatment.

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[2]For this multivariate analysis step, an unweighted mean was used for arriving at a representative score for each school. Since it was thought that using a weighted mean (weighted according to the number of classes in the school) might alter the results, a more expensive second analysis, using the univariate repeated measures approach with weighted means, was carried out. The results were nearly identical (See Table 2-F, Appendix F). Hence, for further analysis, the mean score for each school is determined by the simpler, unweighted, method.

[3]The proportion of incomplete and missing data in the Portland, Oregon, site was so large as to make the supposed randomization process highly suspect. Hence these data were discarded and not used in any of these analyses.

Table 2
Analysis of Variance: Knowledge

Dependent Variables: (1) PIA-Specific Posttest Score
(2) Basic Mathematics Posttest Score

Source	df for Hypothesis	Multivariate Significance Level	Results: Univariate Results[d] PIA-Specific MS	F	p	Basic Math MS	F	p	Direction If Relevant
Mean	1	---	5521.89	---	---	12627.29	---	---	
Among Schools[a]	72	---							
Among Sites	3	.0053*	2.036	0.65	.59	38.82	5.06	.0033*	
Treatment (PIA vs. Control)[b]	1	.009*	26.39	8.39	.0052*	5.043	0.66	.42	PIA higher
Treatment x Site	3	.23	6.189	1.97	.13	9.112	1.19	.32	
Among Schools Within Cells[c]	65	---	3.146	---	---	7.676	---	---	
Within Schools	66	---	---	---	---	---	---	---	
Grades Specific to Sites	4	.55	7.007	1.56	.20	2.525	0.31	.87	
Treatment x Grades Specific to Sites	4	.68	2.452	0.54	.70	7.968	0.97	.43	
Grade x Schools Within Cells	58	---	4.501	---	---	8.240	---	---	

*Statistically significant with 0.05 level as criterion.

[a]Analysis performed using unweighted average of grade means in each school.

[b]Assumes Treatment x Site and Treatment x Grade specific to site effects are negligible but any potential site effects have been eliminated.

[c]Used as error for Among Schools Analysis.

[d]If multivariate test is significant, univariate tests are performed with modified Roy-Bose technique: in this case, the requirement simplifies to use of ordinary F ratios shown.

Table 3
Analysis of Variance: Attitude

Dependent Variables: (1) Enjoyment of Teaching Mathematics
(2) Confidence in Teaching Mathematics

Source	df for Hypothesis	Multivariate Significance Level	Results: Univariate Results — Enjoyment MS	Enjoyment F	Enjoyment p	Confidence MS	Confidence F	Confidence p	Direction If Relevant
Mean	1	---	---	---	---	---	---	---	
Among Schools[a]	72								
Among Cells	7								
Among Sites	3	.17	5.787	81.45	.24	5.650	1.67	.18	
Between Treatments[b]	1	.54	1.468	0.37	.55	1.674	0.49	.48	
Treatment x Site	3	.35	1.598	70.40	.75	1.259	0.37	.77	
Within Cells[c]	65	---	3.980	---	---	3.384	---	---	
Within Cells	65	.0006*	7.611	2.89	.0001*	6.468	1.18	.26	
Within Schools	66								
Grades Specific to Sites	4	.93	1.669	0.63	.64	1.103	0.20	.94	
Treatment x Grades, Specific to Sites	4	.76	2.487	0.94	.44	1.699	0.31	.87	
Grade x Schools (error)	58	---	2.633	---	---	5.482	---	---	

*Statistically significant with 0.05 level as criterion.

[a]Analysis performed using unweighted average of grade means in each school.

[b]After any site effects are removed; assumes Treatment x Site effects are zero.

[c]Used as error for Among Schools Analysis.

For knowledge,[4] the hypothesis of no Site x Treatment interaction was not rejected (Table 2). The F test attained a multivariate significance level of .23, well above the .05 level for clear rejection or the .05-.15 range indicative of moderate evidence against the hypothesis. Using the univariate approach, the significance level for the basic mathematics score was .32, well out of the questionable range; for the PIA-specific score, it was .13, which falls barely within the generally accepted area of moderate negative evidence. Although this in itself was not enough to reject the hypothesis, the residuals (sample values minus predicted values) for the PIA-specific scores were inspected. The largest t ratio for the residuals was -2.04. The effect in this site was the opposite of the effect in the others, but the value was still below the rather liberal Bonferroni critical value of approximately ± 2.39, for 3 df at alpha = .05, and hence is not statistically significant. Thus, for the knowledge variables, the hypothesis cannot be rejected and the simple statistical model is adequate.

For the attitude variables, the results showed even less reason for rejection of the simple model. The multivariate test attained the significance level of .35 and the univariate levels were .75 and .77 for enjoyment and confidence, respectively. The t ratios for all residuals were small. Thus, there is not sufficient evidence to compel us to reject the hypothesis of no Site x Treatment interaction for either knowledge or attitude variables; the simple statistical model is adequate for investigating the main hypotheses stated at the beginning of this section.

Test of the Main Hypotheses

The testing in May, 1971 yielded scores on PIA-specific content and on basic mathematics. Using the simple statistical model with the assumption that the effect of PIA does not vary among grades or sites, both of these knowledge variables were investigated. The multivariate test attained a significance level of .009, well below the necessary .05 and indicative of strong evidence against the null hypothesis (see Table 2). This implies that using PIA in the classroom changed the level of performance on at least one and possibly both of the two variables. The univariate analysis was used to determine which of the variables was thus affected. For PIA-specific knowledge, the F ratio yielded a significance level of .005, well below the .05 required; there is clear-cut evidence against the assumption of zero effect. For basic mathematics, however, the F ratio yielded a significance level of .42, and thus the assumption of no effect on that variable cannot be rejected. PIA changes the level of performance in the target population on PIA-specific knowledge but not, apparently, on other basic mathematics (see Figure 4, p. 21).

However, this apparent lack of effect of PIA on knowledge of basic mathematics was determined using only the posttest scores. A better method of investigation is to use the pretest scores as well, recalling that all items on the posttest were also included in the pretest. Thus a univariate analysis of variance was performed on the gain scores, or differences in score from pretest to posttest. This method is better in the sense that not only is a gain score an unbiased estimate, as is a posttest score alone, but also there is less variance connected with gain scores, and thus results can be obtained with greater precision. In a sense, each teacher acts as his own control when a gain score is used.

The F ratio resulting from this ANOVA reached a significance level of .003, highly indicative that the null hypothesis must be rejected (see Table 2, p. 9). When one investigates gain scores, PIA does affect level of knowledge of basic mathematics. However, the effect is in a negative direction—the gain scores of the target population as a whole are larger than for those who used PIA in the classroom.

When attitudes are analyzed, the results are less striking. Although the observed differences were negative for enjoyment and positive for confidence, neither of these came close to being significant (see Table 3). The attained significance level for the multivariate test was .54 and the univariate levels were .55 and .48 for these two variables.

---

[4]For shorthand convenience, "knowledge" stands for investigations of the hypotheses concerning variables related to scores on pretest and posttest—in the latter case both the PIA-specific test and the basic mathematics. Similarly, "attitude" refers to investigations of hypotheses concerning confidence in and enjoyment of teaching elementary school mathematics, data for which were gathered from the two questionnaires.

Further Analyses

While the analysis could end with what has been discussed so far, there remains the fact that much of the variance in the data came from differences among teachers, and thus among schools, the units of analysis. The precision of various estimates should be increased by eliminating differences associated with pretest scores (for the knowledge analysis) and beginning attitudes as measured on the pre-questionnaire (for the attitudes analysis).

Thus analyses of covariance were also done. For knowledge, the choice of covariates was arrived at somewhat empirically: by determining the set of covariates which reduces the mean square error the most for the least proportionate reduction in degrees of freedom for error (see Table 4). The final set of covariates chosen in this manner was the score on those pretest items which were also on the posttest, dropping the assumption that the regression of posttest scores on pretest scores is the same regardless of whether teachers have used PIA. (This was accomplished by using as a variable the pretest subscore for the experimental group, and zero for the control group.) The use of several other possible covariates was examined, but did not prove fruitful.

The results (see Table 5) agreed with those from analyses of variance reported earlier. The multivariate significance level was .01; therefore the hypothesis that PIA has zero effect on knowledge must be rejected. Further univariate investigation revealed that this non-zero effect showed up on the PIA-specific post-scores, but not on the basic mathematics post-scores (F test significance levels of .045 and .23, respectively). As before, the results showed that using PIA in the classroom significantly increases knowledge related to the PIA content.

Since dropping the assumption of homogeneity of regression (of knowledge scores on pretest basic mathematics score) proved useful in the analysis of covariance, the hypothesis itself was investigated. The multivariate significance level was .08, not below the .05 level for statistical significance, but nonetheless indicative of some evidence against the hypothesis (see Table 5). When this was further investigated, it appeared that there was little difference between experimental and control groups for regression of basic mathematics posttest scores on pretest scores, but there was more apparent effect on the regression of PIA-specific posttest scores on basic mathematics pretest scores (see Figures 2 and 3). The regression coefficient for the experimental group was estimated at .27, while that for the control group was .49. This difference, although still only in the marginal area statistically—the significance level is .09—tends to indicate that among those with low to average scores on the basic mathematics pretest, using PIA makes a marked difference on the PIA-specific posttest when compared with the target population as a whole, whereas among those with pretest scores more than one standard deviation above the mean, the effect is just the opposite: using PIA produces scores relatively lower than in the target population. Restated, the less the knowledge of basic mathematics at the start of the year, the more likely using PIA in the classroom will be effective as a medium of in-service training for PIA content. The same thing is not indicated, however, for learning basic mathematics not specifically in the PIA material.

Although there were gains by both experimental and control groups from basic mathematics pretest to posttest, the gain of the experimental group was estimated to be only about 40% of that of the target population—.79 as compared to 2.00 points, out of a total test score of 20 (see Table 6). Since the standard error of the difference was .39, this difference of 1.21 points is statistically significant. This coincides, as it must, with the finding from the ANOVA reported earlier.

The analysis of covariance of post-experimental attitude, using pre-experimental attitudes as covariates, produced the same results as did the analyses of variance; i.e., no apparent differences between experimental and control groups (see Table 7). Additional analyses of covariance of enjoyment and confidence "gain" scores, using basic mathematics pretest scores as covariates, indicated that there were gains in both enjoyment and confidence over the school year (in the latter, particularly for teachers whose initial basic mathematics scores were high), but that there were no significant differences between experimental and control groups.

Table 4
Determination of Covariate Set for Analysis of Covariance: Knowledge
(Estimated Error Variance with Various Covariate Groups)

Covariates	df for error	Results with Covariates: Mean Square Errors PIA-Spec.	Mean Square Errors Basic Math	Relative to Selected Covariate Group, Proportion Increase (Decrease) in: df for error	Relative to Selected Covariate Group, Proportion Increase (Decrease) in: MSerror for PIA
None	65	3.1363	7.6631	.032	0.511[a]
Pretest Basic Math & Pretest Basic Math x Treatment*	63	2.0754	2.5582	---	---
Total Pretest	64	2.1373	3.0272	.016	0.030[a]
Total Pretest & Total Pretest x Treatment	63	2.1012	3.0159	0	0.012[a]
Pretest Basic Math Only (Pretest 1)	64	2.1393	2.5376	.016	0.031[a]
Pretests 1 & 2	63	2.1423	2.5755	0	0.032[a]
Pretests 1 & 2, Pretest 1 x Treatment	62	2.0687	2.5979	(.016)	(0.003)[b]
Pretests 1 & 2, Pretests 1 & 2 x Treatment	61	2.0801	2.4596	(.032)	0.002[c]

*The selected pair of covariates.

Rationale: Choice of covariates should reduce MSs for error a maximum amount while reduction of df is minimized; or at least obtain optimal combination.

[a]Increase in MS_{error} > increase in df.

[b]Decrease in MS_{error} < decrease in df.

[c]df decreases and MS increases.

In all cases, the MS_{error} for PIA-specific score is criterion.

Table 5
Analysis of Covariance: Knowledge

Dependent Variables: (1) PIA-Specific Posttest Scores
(2) Basic Mathematics Posttest Scores

Source	df for Hypothesis	Multivariate Significance Level	Results: Univariate Results — PIA-Specific MS	PIA-Specific F	PIA-Specific p	Basic Math MS	Basic Math F	Basic Math p	Direction If Relevant
Schools	73								
Mean	1	---		---	---		---	---	
Among Schools	72	---		---	---		---	---	
Covariates (total)[a]	2	.0001*	36.5478	17.61	.0001*	168.4575	65.85	.0001*	
Basic Math Pretest[b]	1	.0001*	71.6221	34.51	.0001*	322.0518	125.89	.0001*	Positive
Basic Math x Treatment[b]	1	.08	6.1639	2.97	.09	1.2535	0.49	.49	Decr. with PIA
Among Cells	7	---		---	---		---	---	
Among Sites	3	.03*	0.8297	0.40	.75	8.1747	3.20	.03*	
Between Treatments[c]	1	.01*	8.7348	4.21	.045*	3.7141	1.45	.23	PIA higher
Treatment x Sites	3	.32	3.8936	1.88	.14	1.1597	0.45	.72	
Residual for Error	63	---	2.0754	---	---	2.5582	---	---	

*Statistically significant at .05 level.

[a]After all site and treatment effects eliminated.

[b]Sum of Squares are "extra" for that particular covariate.

[c]After site and covariate effects eliminated but assuming Treatment x Site interaction effect is zero.

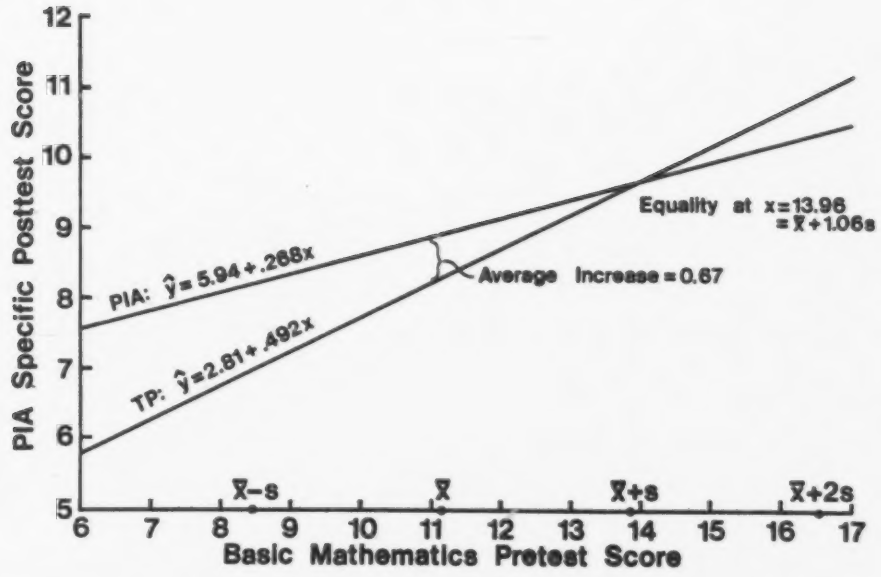


Fig. 2. Regression of PIA-specific posttest scores on basic mathematics pretest scores.

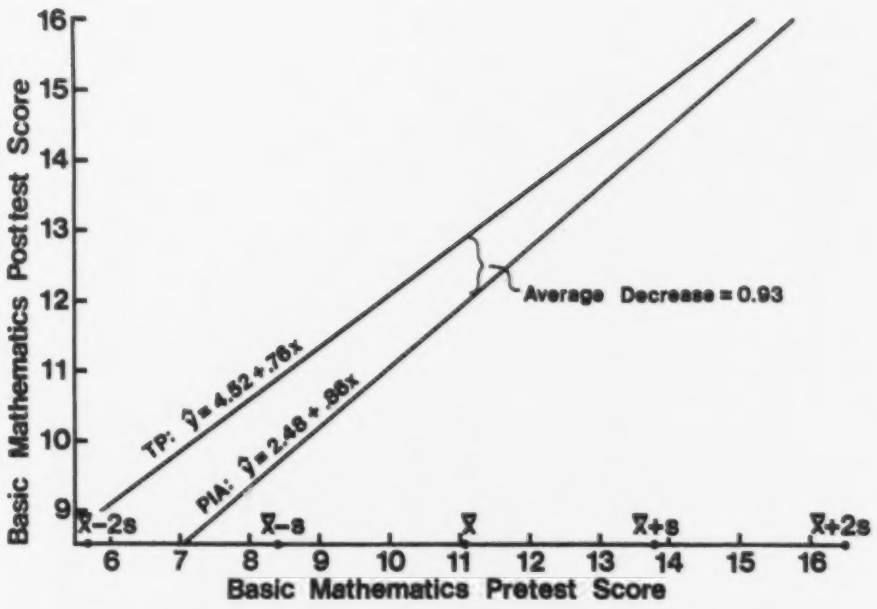


Fig. 3. Regression of basic mathematics posttest scores on basic mathematics pretest scores.

Table 6
Estimated Parameters for Target Population with and without PIA
Least Squares Estimates Assuming Simple Statistical Model

Variable	SITE											
	Urban Site 1				Urban Site 2				Rural Site 1			
	Target Population				Target Population				Target Population			
	With PIA		Without PIA		With PIA		Without PIA		With PIA		Without PIA	
	Est'd Mean	S. E. of Est.	Est'd Mean	S. E. of Est.	Est'd Mean	S. E. of Est.	Est'd Mean	S. E. of Est.	Est'd Mean	S. E. of Est.	Est'd Mean	S. E. of Est.
From Analyses of Variance												
Knowledge												
Pretests												
Subtest 1 (Basic Mathematics)—20 items	a		12.19	.45	a		9.88	.93	a		10.36	.88
Subtest 2 (Other Pretest items)—30 items	a		11.27	.48	a		9.62	.99	a		9.66	.94
Total—50 items	a		23.46	.89	a		19.51	1.82	a		20.02	1.73
Posttests												
PIA-Specific—15 items	9.53	.35	8.32	.36	8.85	.62	7.64	.63	8.94	.60	7.73	.60
Basic Mathematics[b]—20 items	13.28	.49	14.49	.50	9.64	.95	10.86	.96	11.45	.90	12.67	.90
Gain in Basic Mathematics	1.09	.33	2.30	.34	-.24	.58	.98	.60	1.09	.56	2.31	.56
Attitude[c]												
Pre-questionnaire												
Enjoyment	a		8.76	.29	a		7.58	.60	a		7.93	.57
Confidence	a		6.46	.36	a		6.62	.73	a		6.17	.69
Post-questionnaire												
Enjoyment	8.79	.39	9.08	.41	7.36	.70	7.64	.71	8.15	.67	8.44	.67
Confidence	8.39	.36	8.09	.38	7.47	.64	7.16	.66	8.52	.62	8.22	.62
From Analyses of Covariance												
Knowledge												
PIA-Specific[d]	8.95	.31	8.30	.29	9.10	.50	8.46	.53	8.98	.53	8.32	.54
Basic Mathematics (Posttest)[d]	12.49	.34	13.42	.33	10.74	.57	11.66	.60	12.18	.60	13.10	.60
Gain in Basic Mathematics[e]	1.44	.34	2.34	.33	-.32	.56	.57	.60	1.11	.54	2.01	.55
Attitude (Post-questionnaire)												
Enjoyment[f]	8.47	.32	8.73	.33	7.70	.56	7.96	.58	8.35	.53	8.61	.53
Confidence[f]	8.32	.30	7.85	.30	7.57	.52	7.11	.53	8.74	.49	8.28	.49

Table 6 (Continued)

Variable	SITE							
	Rural Site 2				All Sites			
	Target Population				Target Population			
	With PIA		Without PIA		With PIA		Without PIA	
	Est'd Mean	S. E. of Est.	Est'd Mean	S. E. of Est.	Est'd Mean	S. E. of Est.	Est'd Mean	S. E. of Est.
From Analyses of Variance								
Knowledge								
Pretests								
Subtest 1 (Basic Mathematics)—20 items	a		11.97	.70	a		11.10	.38
Subtest 2 (Other Pretest items)—30 items	a		8.48	.74	a		9.76	.41
Total—50 items	a		20.45	1.37	a		20.86	.75
Posttests								
PIA-Specific—15 items	9.08	.49	7.87	.49	9.10	.31	7.89	.32
Basic Mathematics[b]—20 items	13.18	.72	14.40	.72	11.89	.43	13.10	.43
Gain in Basic Mathematics	1.21	.46	2.43	.46	.78	.30	2.00	.30
Attitude[c]								
Pre-questionnaire								
Enjoyment	a		8.56	.45	a		8.21	.49
Confidence	a		6.22	.55	a		6.37	.60
Post-questionnaire								
Enjoyment	8.75	.55	9.04	.55	8.26	.35	8.55	.37
Confidence	7.38	.51	7.07	.51	7.94	.33	7.64	.34
From Analyses of Covariance								
Knowledge								
PIA-Specific[d]	8.66	.10	8.00	.40	8.92	.19	8.27	.29
Basic Mathematics (Posttest)[d]	12.54	.11	13.47	.45	11.99	.22	12.94	.33
Gain in Basic Mathematics[e]	1.53	.46	2.43	.44	.94	.29	1.84	.30
Attitude (Post-questionnaire)								
Enjoyment[f]	8.58	.44	8.84	.44	8.28	.28	8.54	.29
Confidence[f]	7.44	.41	6.98	.41	8.02	.26	7.55	.27

For notes, see next page.

Table 6 (Continued)

[a]Populations "with PIA" but before PIA was used in the classroom are identical to populations "without PIA" because of random assignment to treatments.

[b]Estimates = estimated target population pretest mean + estimated gain (or loss).

[c]For purposes of analysis and comparability, the enjoyment and confidence variables were transformed so that each had a possible range of 1 = low to 13 = high.

[d]Covariates are Basic Mathematics Pretest Score and Basic Mathematics x Treatment Indicator Score.

[e]11.10, the Basic Mathematics Pretest Score mean over all sites, was used as the basis for gain score in each site; covariate is the Basic Mathematics Pretest Score.

[f]Covariates are Enjoyment and Confidence "scores" from pre-questionnaire.

Table 7
Analysis of Covariance: Attitude

Dependent Variables: (1) Enjoyment of Teaching Mathematics
(2) Confidence in Teaching Mathematics

Source	df for Hypothesis	Multivariate Significance Level	Results: Univariate Results: Enjoyment MS	Enjoyment F	Enjoyment p	Confidence MS	Confidence F	Confidence p	Direction If Relevant
Schools	73								
Mean	1								
Among Schools	72								
Covariates (total)	2	.0001*	48.2625	18.75	.0001*	43.4351	20.56	.0001*	
Enjoyment (pre-questionnaire)	1	.0001*	51.3674	19.95	.0001*	7.3895	3.50		
Confidence (pre-questionnaire)	1	.01*	3.1804	1.24		36.3771	17.22	.0001*	
Among Cells	7								
Among Sites	3	.19	1.6080	0.62	.60	5.0761	2.40	.08	
Between Treatments	1	.33	1.2061	0.47	.50	3.8461	1.82	.18	
Treatment x Sites	3	.58	1.0433	0.41	.75	2.5468	1.21	.32	
Residual for Error	63	---	2.5740			2.1126			

*Statistically significant at .05 level.

IV
Summary and Discussion

Perhaps the best way to summarize is to answer the questions posed as the goals of the study (see Figure 4):

1. Do teachers who use PIA increase their knowledge of modern mathematics; i.e., is PIA an effective in-service course?

The answer is a clear yes, provided we restrict ourselves to the general basic mathematics related to the PIA telecasts. However, if we look at content not specifically found in PIA, the answer is no, for the gain scores for non-PIA-specific content, while definitely positive for both groups, were generally higher for the teachers who did not use PIA. It seems that while PIA teachers were gaining in knowledge of PIA content, other teachers made higher gains elsewhere than did the PIA teachers.

2. Do teachers who use PIA become more positively disposed toward teaching mathematics; i.e., enjoy it more or become more confident?

Both groups seemed to make slight gains in enjoyment and larger gains in confidence over the school year, but PIA teachers did not show an advantage over control teachers in this respect.

3 a. Does the effect of PIA on teachers' knowledge vary with the demographic characteristics of the school?

Whereas there was a difference in level of knowledge at the end of the study between teachers in the two urban sites, this difference was not related to use of PIA. PIA was equally effective in both of these urban sites, as outlined in the answer to question 1 above. Nor was any difference found between urban and rural sites; PIA is equally effective as a medium of in-service education in both situations.

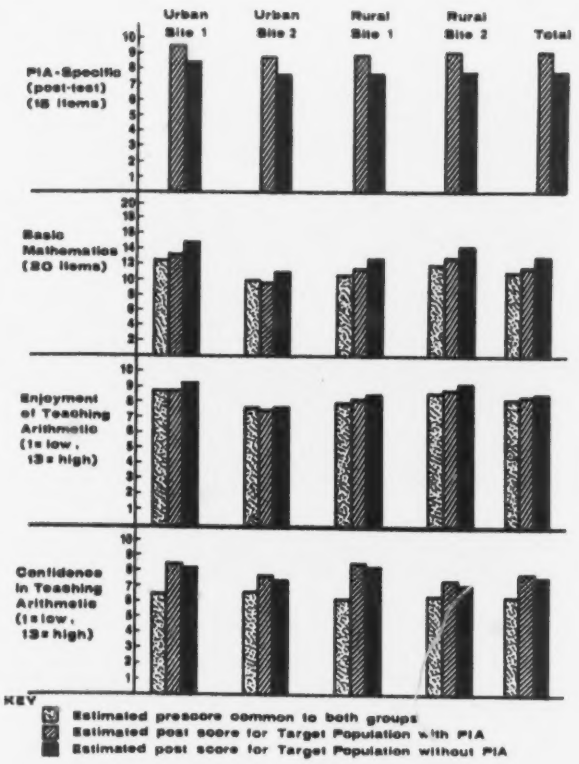

Urban Site 1
Urban Site 2
Rural Site 1
Rural Site 2
Total
PIA-Specific (post-test) (15 items)
Basic Mathematics (20 items)
Enjoyment of Teaching Arithmetic (1= low, 13= high)
Confidence in Teaching Arithmetic (1= low, 13= high)
KEY
Estimated prescore common to both groups
Estimated post score for Target Population with PIA
Estimated post score for Target Population without PIA

Fig. 4. Performance levels on knowledge and attitude variables for target population with and without PIA: Estimated means.

3 b. Does the effect of PIA on teachers' knowledge vary with the grade level taught?

The answer appears to be no.

3 c. Does the effect of PIA on teachers' knowledge vary with the level of teachers' initial general knowledge of basic mathematics?

Here, the answer is a tentative yes: PIA is very effective as an in-service tool for those whose initial level of basic mathematics is low to average; for those whose pretest scores fall more than a standard deviation above the mean, the effect is lessened.

4. Does the effect of PIA on teachers' attitudes vary with these same three factors?

The demographic chracteristics of the school seem to make no difference, nor does the grade level taught. Whereas there was a slight increase in teachers' enjoyment no matter what their initial level of knowledge, those who scored high on the basic mathematics pretest seemed to have greater gains in confidence than those whose initial scores were lower; however, this occurred regardless of whether PIA was used.

GPO 827-794—3

Appendix A
Overview of the Key Mathematical Ideas in Patterns in Arithmetic, Grades 1–6

The following is a brief description of the fundamental concepts which run through the *Patterns in Arithmetic* series.

1. Set: The concept of a set is fundamental for developing and communicating ideas in mathematics. Since the idea of a set is a rather primitive one, as early as Grade 1, pupils become familiar with the simple concepts which involve sets, such as setting up a one-to-one correspondence (matching) between sets and comparing the numerousness of two sets. The set idea is found to be particularly useful in the teaching of addition (set union) and subtraction (set complementation). Set ideas are also applied in the later grades to equivalent fractions, equivalent ratios, and geometric figures.

2. Number: One of the main features of the program is the logical development of the rational numbers, beginning with the positive integers. Near the beginning of Grade 3 the pupils will be able to determine the cardinality of any set with less than 10,000 members and to use the ordinal numbers. During the First, and again the Second Grade, the pupil is taught the ideas of betweenness, less than, greater than, and equal to for numbers less than 1,000. The number line is used to picture the order of integers and the pupil is taught how to use this line to perform addition and subtraction. These ideas are reinforced and extended so that near the end of Grade 6 the pupil is totally familiar with the positive rational numbers. Other miscellaneous topics covered in the series are even and odd numbers, prime numbers, prime factorization of natural numbers, the least common multiple and greatest common factor of pairs of natural numbers, and the negative integers.

3. Numeration System: After some degree of understanding has been achieved in counting with both cardinal and ordinal numbers, the pupils are taught how to write the numbers 0–9, and after 9, the concept of place value is introduced. Although the pupils do not fully comprehend place value at this time, extensive use of tally charts and regrouping is introduced in order to overcome some of the mystery of place value. Near the beginning of Grade 4 the pupil should be able to write all numerals and interpret place value for larger numbers. Although emphasis is on the decimal system (Base 10), numeration systems with other bases (Base 5, Base 2) are introduced to pro-

mote understanding of place value. Positive rational numbers and the several notations associated with them (common fraction, mixed number, decimal fraction) are introduced in Grades 4–6.

4. Operations: A considerable portion of elementary school arithmetic is concerned with the four fundamental operations: addition, subtraction, multiplication, and division; and how these operations act with various sets of numbers (natural numbers, positive rationals, integers). Of course, one of the major objectives of any elementary school arithmetic program is to develop accuracy and speed in computing. However, computing in itself is not enough; therefore, we have placed considerable emphasis on ideas associated with computing in an attempt to make the four operations more than rote calculation. For example, in forming the sum 3 + 2, the pupil considers a stationary set with 3 objects and another set of 2 objects which appear to be joining the given 3. Thus 3 + 2 is looked upon as a set of 3 being joined by a set of 2 to form a set of 5. The concept of column addition is approached through tally charts in an attempt to impart some understanding of place value in our Base 10 numeration system. Rectangular arrays are used to analyze geometrically the operation of multiplication. Geometric arguments are made to lend substance to the operations applied to the positive rationals. The notion of operation is extended in Grades 5 and 6 by introduction of an operation with a simple geometric realization.

   Beginning in the First Grade the commutative, associative, and distributive properties of the operations receive considerable attention as aids to computation. Not until Grade 5 are the properties formalized for the pupils. Upon completion of Grade 6 the pupil will be able to compute efficiently with the positive rational numbers in their many notations.

   Throughout PIA the pupils are presented verbal situations in which they can apply their newly acquired computational skills. In these situations special attention is devoted to developing the ability to formulate mathematical sentences in a clear and natural way.

5. Mathematical Sentence (Equation): Since language is the vehicle through which we communicate our ideas, it is important that pupils begin to develop the ability to generate mathematical sentences as soon as possible. In the First Grade, the pupil encounters many experiences with pictures and objects which lead to basic sentence forms. In word problems pupils are requested to "write a sentence which tells the story of the problem" before finding the solution. (Problem: A box holds 6 apples. How many boxes are needed for 30 apples? Sentence: N x 6 = 30.) Throughout the series the same importance is attached to the need to translate verbal problems into mathematical sentences.

6. Measurement: Owing to its importance in everyday life as a key link between our physical and social environment, we begin a systematic study of measurement in the First Grade. As an introduction, the First Grade pupil becomes familiar with the concept of relative length (the desk is less than five pencil-lengths long) and a non-standard unit of measurement (the pencil above). Upon completing the first four grades, the pupil will be able to carry out approximate linear measurements in standard units (inches, feet) and he will be able to find the perimeter of some elementary geometrical forms (triangle, rectangle). In Grades 5 and 6 the concepts of area and volume are discussed in terms of non-standard and standard units of measure and approximation techniques are used to estimate areas and volumes of irregular regions. Formulas for areas or volumes of elementary regions, such as a rectangular region and right rectangular prisms, are introduced.

7. Geometry: One of the more unique features of the program is a systematic development of elementary geometrical concepts beginning in the First Grade. Aside from learning the names of the more common geometrical figures, the pupil becomes familiar with open and closed curves, interior and exterior of geometrical forms, points, lines and angles, intersection of curves, parallelism, and perpendicularity. The notions of congruence and similarity are introduced intuitively in the early grades and in the later grades approached from transformations in the plane (reflections and dilations).

8. Practical Aspects: Although the practical aspects should not perhaps be mentioned as a fundamental strand, these aspects are important enough in the daily activities of pupils to warrant special attention. Beginning in the primary grades the pupil is exposed to aspects of linear and cubic (cup, pint, etc.) measurement. By the end of Grade 4 the pupil will have some experience in measuring the boundary and area of plane geometrical forms. Other practical aspects covered in the first three grades are money and making change and use of the thermometer.

Appendix B
Pre-Questionnaire—September, 1970

TEACHER INFORMATION QUESTIONNAIRE

(Do not mark in this space)
School Code
_____

1. School:_____ 2. City:_____

3. Your age:_____ 4. Sex: M_____ F_____

5. How many years have you taught?(Include this year but not practice or intern teaching.) _____

6. What grade do you teach? (check one)

First _____
Second _____
Third _____
Fourth _____
I do not teach one grade, but am an arithmetic/mathematics specialist in several grades. _____

7. How many years have you taught this grade or in this capacity? _____

8. Prior to this school year, have you ever used a televised arithmetic/mathematics program at least once a week in your classroom?

Yes _____
No _____

9. Number of semesters of mathematics courses taken in high school: _____

10. Number of semesters of mathematics courses taken in college (undergraduate or graduate):

Mathematics for elementary school teachers _____
College algebra _____
Other (e.g. trigonometry, calculus, beyond calculus) _____

11. Number of semesters of mathematics education courses taken (undergraduate or graduate):

Methods of teaching arithmetic/mathematics _____
Seminars in mathematics education in elementary schools _____

12. When did you complete the following (give year for all which apply):

High School 19_____

Bachelor's 19_____

Master's 19_____

Doctorate 19_____

13. Have you received college credit for course work taken since your last degree?

Yes _____

No _____

If "yes":

How many credits or points _____

In what year did you last receive such credit? 19_____

14. Have you ever attended a National Science Foundation or other institute in modern mathematics?

Yes _____

No _____

If "yes":

How many college credits did you receive for the institute? _____

In what year did you last attend such an institute? 19_____

15. Have you ever attended an inservice course in modern arithmetic or mathematics sponsored by your school or school system?

Yes _____

No _____

If "yes":

In what year did you complete the inservice course(s)? 19_____

Approximately how many hours were spent in the inservice course(s) (check one):

1-4 hr___ 5-9 hr___ 10-19 hr___ 20-40 hr___ more than 40 hr___

16. Compared to the other subjects you teach, how do you enjoy teaching arithmetic? (check one)

more than any other	better than most	moderately well	less than most	least of all
_____	_____	_____	_____	_____

17. When teaching the more difficult concepts of arithmetic, how confident do you feel about your knowledge of those concepts?

completely confident	quite confident	somewhat confident	not very confident
_____	_____	_____	_____

31/32

Appendix C
Post-Questionnaire—May, 1971

TEACHER QUESTIONNAIRE

(Do not mark in this space)
Teacher Code
_____

1. Name: _____

2. School: _____ 3. City: _____

4. What grade do you teach? (check one)

First _____

Second _____

Third _____

Fourth _____

I do not teach one grade, but am an arithmetic/ mathematics specialist in several grades. _____

5. Have you taught the class you are now teaching for this entire school year? If no, please explain: Yes_____ No_____

_____

_____

6. Compared to the other subjects you teach, how do you enjoy teaching arithmetic? (check one)

more than any other	better than most	moderately well	less than most	least of all
_____	_____	_____	_____	_____

7. When teaching the more difficult concepts of arithmetic, how confident do you feel about your knowledge of those concepts? (check one)

completely confident	quite confident	somewhat confident	not very confident
_____	_____	_____	_____

8. How many Patterns in Arithmetic telecasts did your class view this year? (check one)

all (except for those shown during vacation periods)	most	about half	a few	none
_____	_____	_____	_____	_____

9. How many pages in the PIA student exercise book did your class complete? (check one)

all	most	about half	a few	none
_____	_____	_____	_____	_____

PIA school teachers: go on to page 2, question #10.

Control school teachers: skip to page 5 explanation of test.

10. Did you learn any new instructional methods from the Patterns in Arithmetic series?

many _____

some _____

none _____

11. Did you learn any new mathematics concepts from the PIA series?

many _____

some _____

none _____

12. For how many lessons did you read the Mathematical Background section of your PIA teacher's manual?

all _____

most _____

about half _____

a few _____

none _____

13. Now that you have had the experience of teaching with PIA for one year, how do you think things would go if you used the program again next year? (Check the response which comes closest to how you feel.)

This year went well, but next year would be even better _____

This year went well, and next year would be about the same _____

This year did not go too well, but next year would definitely be better _____

This year did not go too well, and next year would be about the same _____

14. How do you feel about the effect PIA this year has had on your pupils' understanding of arithmetic?

Better than other classroom methods _____

Positive, but not necessarily better than other methods _____

Negative or no effect _____

(I have no basis to judge since this was my first year teaching) _____

15. Was it possible for you to individualize instruction using the PIA series?

all of the time _____

much of the time _____

some of the time _____

never or rarely _____

16. Was Patterns in Arithmetic your major (basic) arithmetic program or a supplemental program this year? (check one)

major program _____

supplemental program _____

I used PIA rarely or not at all _____

17. If PIA were available to you next year, how would you use it? (check one)

major program _____

supplemental program _____

I would use it rarely or not at all _____

Directions: Please rate each aspect of PIA listed below by placing a check in one space.

EXAMPLE

Television reception

poor |_____|_____|_____✓_____|_____| good

Value of the Patterns in Arithmetic teacher's manual

useless |_____|_____|_____|_____| useful

Value of the pupil exercise books

useless |_____|_____|_____|_____| useful

Value of the telecasts for students

useless |_____|_____|_____|_____| useful

Attitude of students toward telecasts

disinterested ——— interested

Quality of the TV Teacher's presentations

poor ——— good

Pacing of the TV Teacher's presentation

too fast ——— too slow

just right

Time interval between telecasts

too short ——— too long

just right

Level of difficulty of PIA for your class

too hard ——— too easy

just right

Your attitude toward PIA at the beginning of the school year

negative ——— positive

Your attitude toward PIA now

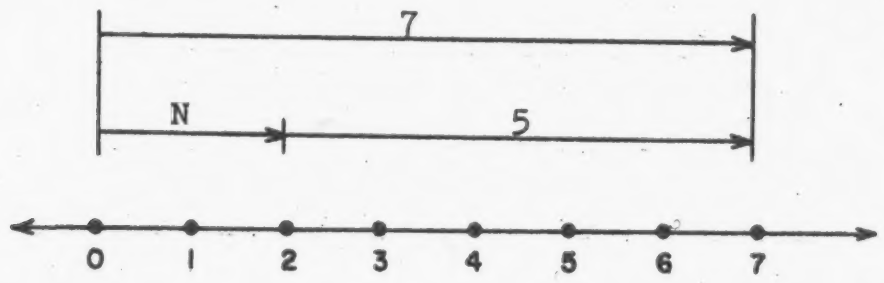
negative ——— positive

GPO 827-794—4

Appendix D
Pretest—September, 1970

Note: Questions with an asterisk also appeared on the posttest.

1.* Which of the following number sentences are equivalent to the sentence whose model is shown on this number line?



(a) $N = 7 + 5$
(b) $N = 7 - 5$
(c) $N + 5 = 7$
(d) $7 - N = 5$

A. Only (a) and (b)
B. Only (b)
C. Only (b) and (c)
D. Only (b), (c) and (d)
E. All of these.

2.* If x and y are positive even integers and if p and q are positive odd integers, which of the following is FALSE?

A. $x + y$ is a positive even integer
B. $p + q$ is a positive odd integer
C. xy is a positive even integer
D. pq is a positive odd integer
E. px is a positive even integer

3. Which of the following are true?

(a) $\frac{3}{7} \times \frac{2}{5} = \frac{5}{35}$

(b) $\frac{18}{35}$ is the reciprocal of $\frac{35}{18}$

(c) $\frac{4}{5} \times \frac{7}{5} = \frac{11}{5}$

(d) $\frac{0}{7} \times \frac{5}{7} = \frac{5}{49}$

A. Only (a)
B. Only (b)
C. Only (a) and (b)
D. Only (c) and (d)
E. All of these.

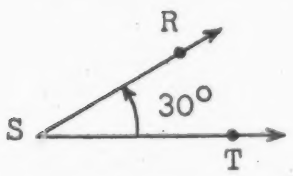
4. In what base are the numerals written if $2 \times 2 = 10$?

A. Base four
B. Base three
C. Base two
D. Base five
E. None of the above is correct.

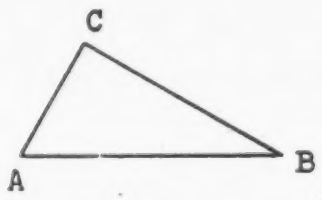
5. The equality of the arrays above illustrates which of the following properties?

A. Commutative property of addition
B. Commutative property of multiplication
C. Associative property of addition
D. Associative property of multiplication
E. Distributive property of multiplication with respect to addition

6. Given $\angle RST$ as the unit angle



and the $\triangle ABC$



The sum of the measures of the angles of $\triangle ABC$ in terms of the unit angle is:

A. 6 units
B. 7 units
C. 5 units
D. 180 units
E. None of these.

7. A ball club won 4 of the 8 games already played. If it wins the next two games, what percent of the games will it then have won?

A. 80
B. 70
C. 75
D. 50
E. 60

42

8. Which of the following represents the same number as $(6 \times 10^4) + (6 \times 10^2) + (6 \times 10^1)$?

A. 6,066
B. 6,660
C. 60,066
D. 60,660
E. 66,600

Questions 9 and 10 refer to the addition exercise below:

$$\begin{array}{rl} & 672 = 6 \text{ hundreds} + 7 \text{ tens} + 2 \text{ ones} \\ + & \underline{834 = 8 \text{ hundreds} + P \text{ tens} + 4 \text{ ones}} \\ & Q \text{ hundreds} + 10 \text{ tens} + 6 \text{ ones} \\ & = R \text{ thousands} + S \text{ hundreds} + T \text{ tens} + U \text{ ones} \\ & = R, S\ T\ U \quad \text{(Final answer)} \end{array}$$

9.* P =

A. 8
B. 3
C. 83
D. 34
E. 834

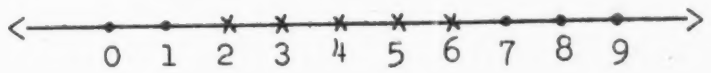
10.* In the final answer, R, S T U represents:

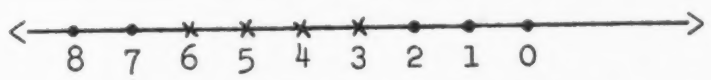
A. The product of the numbers R,S, T and U.
B. The sum of the numbers R, S, T and U.
C. $(R \times 1000) + (S \times 100) + (T \times 10) + (U \times 1)$.
D. $(R \times 1) + (S \times 10) + (T \times 100) + (U \times 1000)$.
E. Eleven thousand, five hundred six.

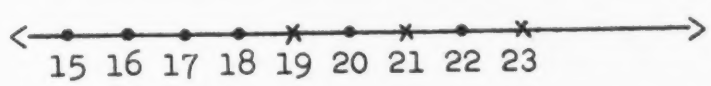
11.* In order to find the product 6×9 we might:

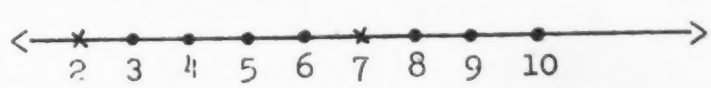
A. Form the union of two disjoint sets with six and nine members.
B. Form a six by nine array.
C. Use the division algorithm.
D. Notice that $(6 \times 9)_{\text{base 10}} = 3_{\text{ten}} \times (6_{\text{three}} \times 9_{\text{three}})$.
E. Draw a triangle with sides 6" and 9" long.

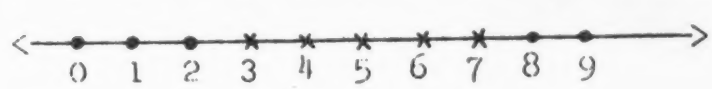
12. Which of the following sets of points best represent the inequality $2 < n < 7$ where n is a <u>whole</u> number?

A. 

B. 

C. 

D. 

E. 

$$\begin{array}{r} 5 \\ \times\ 4 \\ \hline 20 \end{array}$$

13.* Which of the following checks of the computation above makes direct use of the commutative property?

A. $4\overline{)20}^{\,5}$ $\quad\underline{20}$ $\quad 0$

B. $5\overline{)20}^{\,4}$ $\quad\underline{20}$ $\quad 0$

C. $5 + 5 + 5 + 5 = 20$

D. $4 + 4 + 4 + 4 + 4 = 20$

E. $4 \times 5 = 20$

$$4 \times 376 = (4 \times 300) + \square + (4 \times 6)$$

14.* What number goes in place of $\square$ above?

A. 12

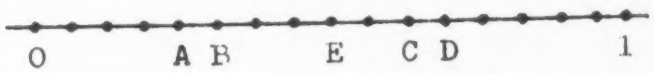
B. 24

C. 28

D. 240

E. 280

15.* Which of the points A, B, C, D, or E on this number line can be named by the fraction $\frac{5}{8}$?



16. 6120_{nine} is how many times as large as 612_{nine} ?

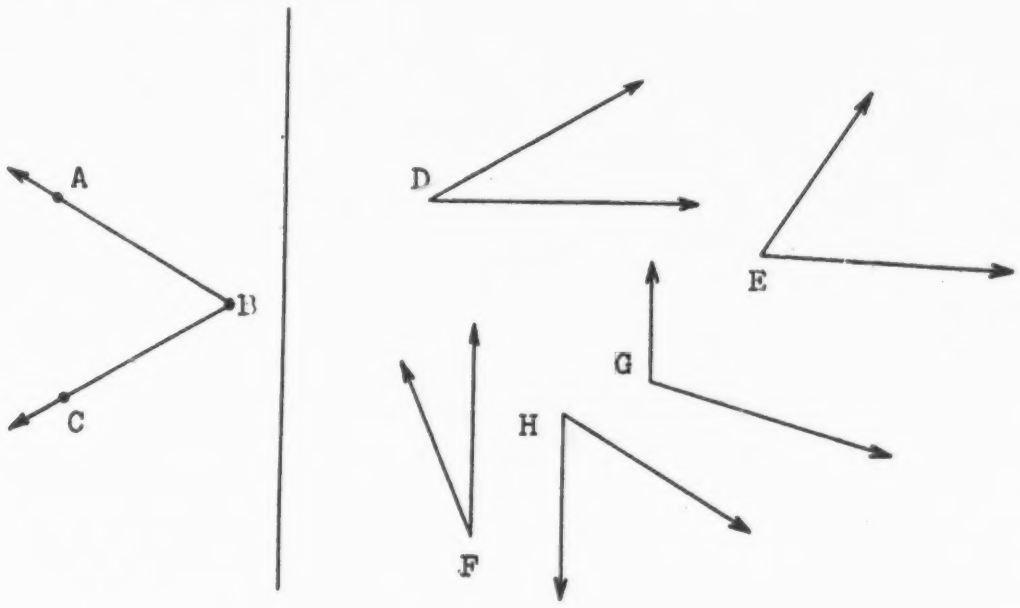
A. twelve
B. ten
C. nine
D. five
E. None of the above is correct.

17. Indicate which English sentence or sentences correspond to the open sentence, $K < 13$.

(a) The temperature is below thirteen degrees.
(b) Joan bought more than thirteen pairs of shoes.
(c) There are fewer than 13 books overdue.
(d) Bill has thirteen dollars.

A. Only (a) and (c)
B. Only (c) and (d)
C. Only (a), (b), and (c)
D. Only (b), (c) and (d)
E. All of them.

18. Which of the following angles appear to be congruent to $\angle ABC$?

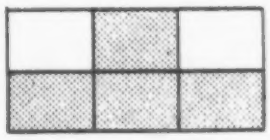


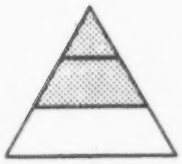
A. Only $\angle D$ and $\angle F$
B. Only $\angle E$ and $\angle H$
C. Only $\angle D$ and $\angle G$
D. Only $\angle D$, $\angle E$, and $\angle G$
E. None of these.

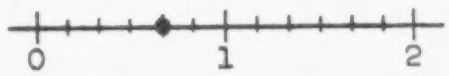
45

19. Which of the following are models for the same rational number as this one?

(a)

(b) 

(c) 

(d) 

A. Only (a)
B. Only (c)
C. Only (a) and (b)
D. Only (a), (b) and (d)
E. All of these

20. Which of the following are true?

(a) If $\frac{1}{3} - r = \frac{3}{4}$, $r = \frac{5}{12}$

(b) $(\frac{8}{2} - \frac{3}{2}) - \frac{1}{2} = \frac{8}{2} - (\frac{3}{2} - \frac{1}{2})$

(c) $\frac{3}{4} - \frac{1}{3} = \frac{1}{3} - \frac{3}{4}$

(d) $\frac{4}{5} - \frac{0}{5} = \frac{0}{5}$

A. Only (a)
B. Only (b)
C. Only (a) and (c)
D. Only (b) and (d)
E. None of these.

21. The least common multiple of 18, 27, and 45 is:

A. 3×3
B. $2 \times 3 \times 5$
C. $2 \times 3 \times 3 \times 3 \times 5$
D. $2 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 5$
E. None of the above.

Tom

$7\overline{)598}\,70$

Mary

$7\overline{)598}\,80$

Dick

$7\overline{)598}\,8$

22. The algorisms above by different pupils are based on interpreting division as successive subtraction. Which of the following statements can be made about the step ccmpleted by the pupils?

A. All are wrong

B. Only Mary is right

C. Only Tom and Mary are right

D. All are right, but Dick's technique is the most efficient

E. All are right, but Mary's technique is the most efficient

23. Four of the following numerals are names for the same number. Which one names a different number?

A. 0.025

B. 2.5%

C. $\frac{1}{40}$

D. $\frac{25}{100}$

E. 25 thousandths

24. What is the hundreds digit in the answer to 3375.49 - 861.1?

A. 5

B. 2

C. 4

D. 1

E. 9

25. What is the number of <u>different</u> prime factors of 60?

A. 0

B. 1

C. 2

D. 3

E. 4

26. Choose the best way to complete this statement: Standard units of measurement are used because

A. it is important for people to use the same unit in dealing with each other.

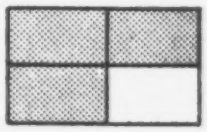
B. standard units give more accurate measurements than units which are not standard.

C. people have always used them.

D. they are all related to base 10 numeration.

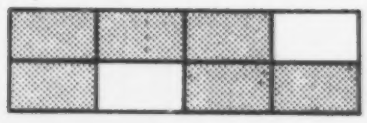
E. the English system is more natural than the metric system.

27. Which of the following are models for the same rational number as this one?

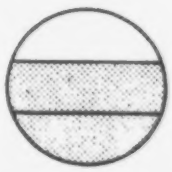


(a)

(b)



(c)



(d)

A. Only (a) and (b)
B. Only (a)
C. Only (d)
D. Only (a), (b) and (c)
E. All of these.

$$
\begin{array}{r|l}
3\overline{)27} & \\
\underline{12} & 4 \\
15 & \\
\underline{m} & k \\
9 & \\
\underline{9} & 3 \\
0 & 9
\end{array}
$$

28.* If the algorism above is based on interpreting division as successive subtraction, then k =

A. 2
B. 3
C. 5
D. 6
E. 9

29. Which of the following are empty sets?

(a) All odd numbers exactly divisible by 2.

(b) Women who have been president of the U.S.A.

(c) All positive even numbers exactly divisible by 5.

(d) All whole numbers which do not have 1 as a factor.

A. Only (b)

B. Only (a) and (b)

C. Only (b) and (c)

D. Only (a), (b) and (d)

E. All of these.

30. Consider the set $\{p, q, r, s, t\}$ and the operation ϕ between two members of this set. The following is a table for ϕ on this set.

ϕ	p	q	r	s	t
p	p	p	p	p	p
q	p	q	r	s	t
r	p	r	t	q	s
s	p	s	q	t	r
t	p	t	s	r	q

Which of the following statements is true?

I. $r \phi s = s \phi r$

II. $r \phi (t \phi s) = (r \phi t) \phi s$

III. p is the identity for ϕ

A. I only

B. II only

C. III only

D. I and II only

E. I, II, and III

31.* Suppose $\_\_|\_\_$ is a binary operation with

$1 \_\_|\_\_ 1 = 0$ $\quad$ $7 \_\_|\_\_ 2 = 13$

$2 \_\_|\_\_ 2 = 3$ $\quad$ $4 \_\_|\_\_ 4 = 15$

$5 \_\_|\_\_ 6 = 29$ $\quad$ $9 \_\_|\_\_ 2 = 17$

What is $6 \_\_|\_\_ 3$?

A. 6

B. 3

C. 9

D. 17

E. 18

$$\begin{array}{r} 432 \\ \times \quad 2 \\ \hline 4 \\ 60 \\ 800 \\ \hline 864 \end{array}$$

32. The basic property used in the algorism above is illustrated by which of the following?

A. $x(yz) = (xy)z$

B. $x + (y + z) = (x + y) + z$

C. $x(y + z) = xy + xz$

D. $x + y + z = z + y + x$

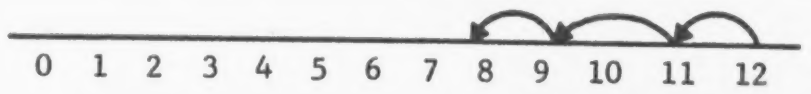
E. $xyz = zyx$

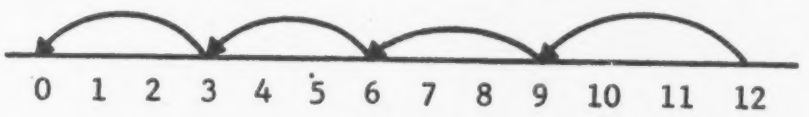
33. The most important common property of the sets {A, 2,* } and {0, 1, 2} is suggested by

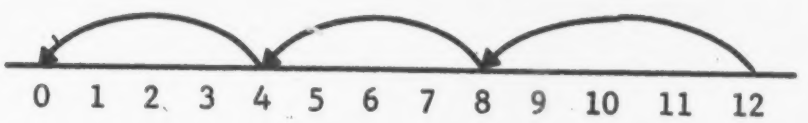
A. 2
B. {,,}
C. 6
D. empty
E. 3

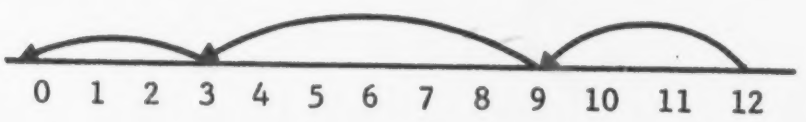
$$\begin{array}{r|l} 3\,)12 & \\ \underline{3} & 1 \\ 9 & \\ \underline{6} & 2 \\ 3 & \\ \underline{3} & \underline{1} \\ 0 & 4 \end{array}$$

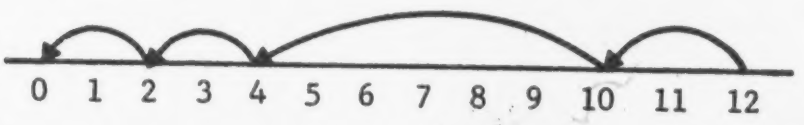
34.* Which of the following number-line diagrams illustrates the subtraction operations depicted in the division algorism above?

A.


B.


C.


D.


E.


35.* The inverse operation for addition is:

A. addition.
B. subtraction.
C. multiplication.
D. division
E. None of these.

36. What is the sum of 6_{eight} and 3_{eight} ?

A. 9_{eight}
B. 18_{eight}
C. 10_{eight}
D. 11_{eight}
E. None of these.

$$12 + 3 = 4$$

37. If the division above is interpreted as finding how many groups of 3 each make up 12, which of the following operations is most nearly analogous to this?

A. Finding the size of a hand of cards if you know the size of the deck and the number of hands
B. Finding the area of a lot if a tract of land of known area is subdivided into a given number of equal lots
C. Finding the number of feet in a 48-inch length with a 12-inch ruler
D. Dividing a pie equally among a given number of people
E. Finding what number can be subtracted four times from 12

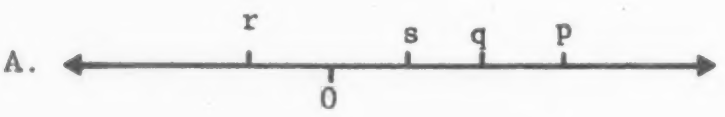
38.* A point P lies 4.03 inches from the center of a circle with a radius of 4.3 inches. Which of the following is true?

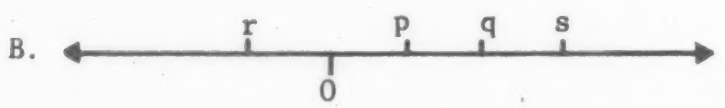
A. P lies outside the circle
B. P lies at the center
C. P lies inside the circle, but not at the center
D. P lies somewhere on the circumference
E. It cannot be determined whether P lies inside, on, or outside the circle

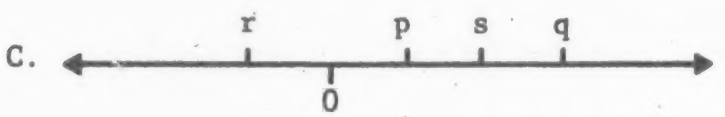
39. Which of the following is the best example of something unique?

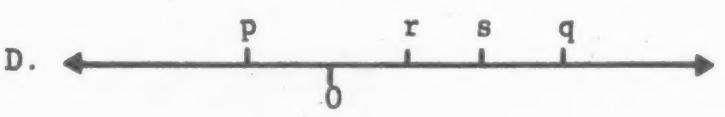
A. a symbol for the number of fingers you have
B. the second house from a certain corner in a residential district
C. the number of eggs in a dozen
D. a point on a ruler which is one inch from the three-inch mark
E. a rectangle with one side five inches long.

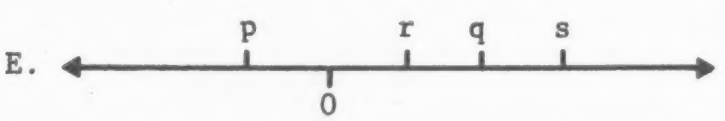
40.* If $r < 0$, $s > p$, $s < q$, and $0 < p$, which of the following number lines shows the correct relative positions of p, q, r, and s?

A. 

B. 

C. 

D. 

E. 

41. If 13 candy bars are to be given to 3 boys so that each boy receives at least one candy bar, which of the following statements must be true?

A. One boy receives 10 candy bars.
B. Each boy receives at least 2 candy bars.
C. One boy receives at least 5 candy bars.
D. One boy receives at least 7 candy bars.
E. The boys all receive a different number of candy bars.

42. If $X = \{b, a\}$ and $Y = \{7,8\}$, the most important property of the set $X \cup Y$ is suggested by

A. $\{\ \}$
B. 7
C. 4
D. the order b, a, 7, 8
E. 2

43.* If x, y, z are numerals in <u>base five</u> notation, which of the following are correct?

(a) $x + y = y + x$
(b) $x(y + z) = x \cdot y + x \cdot z$
(c) if $x < y$ and $y < z$ then $x < z$:

A. Only (a)
B. Only (b)
C. Only (a) and (b)
D. Only (a) and (c)
E. (a), (b), and (c)

44. Which of the following are true?

(a) $\frac{2}{3} = \frac{4}{6} = \frac{8}{12} = \frac{10}{15} = \frac{14}{21}$

(b) $\frac{2}{3} + \frac{1}{4} = \frac{11}{12}$

(c) $\frac{5}{24} + \frac{7}{18} = \frac{43}{72}$

(d) $\frac{2}{3} + \frac{5}{7} = \frac{28}{21}$

A. Only (a)
B. Only (a) and (b)
C. Only (c) and (d)
D. Only (a), (b) and (c)
E. All of them.

45.* If A and B are two sets which can be matched (put in 1 to 1 correspondence) then:

A. A and B must both be sets of numbers.
B. A and B must have the same number of elements.
C. Neither A nor B can be the empty set.
D. A and B must both be sets of points.
E. A and B must each have one member.

46.* Which of the following is the multiplicative inverse of 4.0?

A. 0.04
B. 0.25
C. 0.4
D. 0.5
E. 1.25

47.* Which one of the following division examples will result in the quotient 25.2 ?

A. $5\overline{)\ .126}$
B. $.5\overline{)\ .126}$
C. $.05\overline{)\ .126}$
D. $.005\overline{)\ .126}$
E. $.0005\overline{)\ .126}$

48.* A football team starts the following series of plays from its own 20-yard line: gains 8 yards, loses 2 yards, loses 1 yard, gains 6 yards. On what yard line is the ball now?

A. 11
B. 26
C. 31
D. 34
E. 37

49. Which of the following intersection sets is <u>impossible</u> for a straight line and a triangle?

A. The empty set
B. A set of exactly one point
C. A set of exactly two points
D. A set of exactly three points
E. A set of an infinite number of points.

GPO 827-794—5

50. The equality 2 x (3 + 4) = 2 + (3 x 4) is an illustration of which of the following principles?

A. The associative law of addition

B. The commutative law of addition

C. The distributive law of multiplication with respect to addition

D. The commutative law of multiplication

E. None of these

Appendix E
Posttests—May, 1971

Note: Questions 1-15 were, for the most part, different for each grade level. These appear in this appendix as PIA-specific content, Grades 1, 2, 3, and 4. Questions 16-35 were the same for all grade levels, as they were culled from the common pretest. In the actual tests, there was no break between questions 15 and 16. However, to avoid duplication, questions 16-35 appear only once in this appendix, under the label Basic Mathematics.

EXPLANATION OF THE TEST

This is a test on some of the basic mathematical concepts which underlie a modern arithmetic program. Many of these concepts have not, in the past, been stressed in the elementary school curriculum; consequently, some of the ideas in this test may be unfamiliar to many teachers.

Results from this test will be compared with those from a similar test given last fall. If you did not take a test similar to this last fall, check here:

I did not take a test similar to this one last fall. ☐

DIRECTIONS

All of the questions in this test are multiple choice. You are to select the one best answer from the five choices given and circle the letter of that answer choice.

Here are two sample questions:

EXAMPLE 0.

Which of the following is a multiple of 10?

A. 1
B. 5
C. 101
D. 8460
E. 77777

The correct answer is D , since 8460 = 846 x 10. Therefore, the letter D would be circled:

A. 1
B. 5
C. 101
(D.) 8460
E. 77777

EXAMPLE 00.

Which of these is another name for 27?

(I) 2 x 10 + 7

(II) 3 x 3 x 3

(III) 72

A. Only I

B. Only III

C. Only I and II

D. Only I and III

E. I, II and III

Both 2 x 10 + 7 and 3 x 3 x 3 are equal to 27. Therefore the correct answer is "C. Only I and II," and the letter C is circled.

You may use any extra space for scratch work. Go ahead and begin.

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PIA-specific content: Grade 1

1. Which of the points are on the same side of the curve?

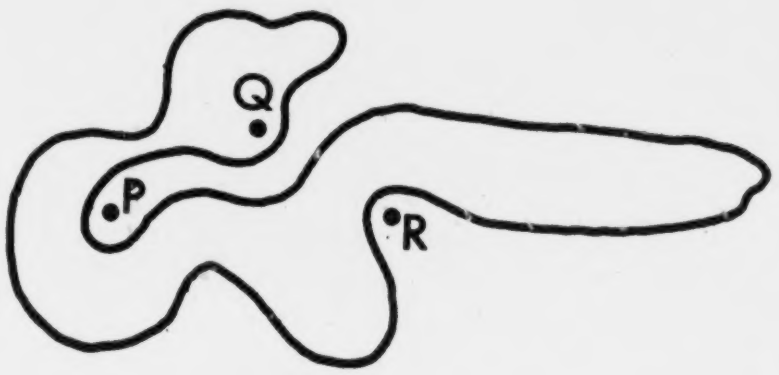
A. P and Q

B. P and R

C. R and Q

D. P, Q and R

E. none of them



2. In which of the following pairs of numerals does the '8' have the same place value?

(I) 81 and 812 (II) 4802 and 876 (III) 8 and 538

A. only I

B. only II

C. only I and II

D. only I and III

E. only II and III

3. Set A has members 2, 3, 4, 5, and 6. Which of the following numbers comprise a subset of A?

A. 0, 1, 7, 8, 9

B. 1, 2, 3, 4, 5

C. 1 2

D. 2, 5

E. 0, 1

4. Which is an example of changing something from a closed curve to an open curve?

A. breaking a rubber band

B. peeling a tangerine

C. bending a hula hoop

D, cutting a hole in a fish net

E. snapping a rung in a 6-foot ladder

5. If set A = $\{1, 2, 3\}$ and set B = $\{3, 2, 1\}$, then A and B are

A. equal

B. equivalent but not equal

C. disjoint

D. associative

E. commutative

6. Which point is inside the curve?

A. A

B. B

C. C

D. D

E. none of these

7. Which of these shows the meaning of inverse operations?

A. $3 - 3 = 0$

B. $3 + 0 = 3$; $3 - 0 = 3$

C. $3 + 5 = 5 + 3$

D. $(3 + 5) - 5 = 3$

E. $(3 + 5) + 0 = 3 + (5 + 0)$

8. Which of these is an example of the commutative property of addition?

A. $6 + 0 = 6$

B. $2 + 2 = 2 \times 2$

C. $2 + 6 = 6 + 2$

D. $2 + 6 = 3 + 5$

E. $6 + (2 + 3) = (6 + 2) + 3$

9. If a, b, c, d and e are whole numbers, and if $a > d$, $b < c$, $e > c$ and $a < b$, which is the largest number?

A. a

B. b

C. c

D. d

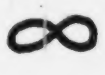
E. e

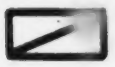
10. Which of the following is a simple closed curve:

A.

B.

C.

D. 

E. 

11. Which of the following is exact? Finding out the number of

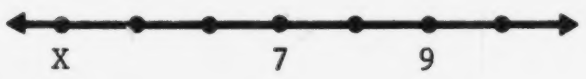
A. pounds a boy weighs

B. inches in a foot

C. cups of water in a bowl

D. miles between two cities

E. square inches in the area of this page

12. 

On the number line above, X =

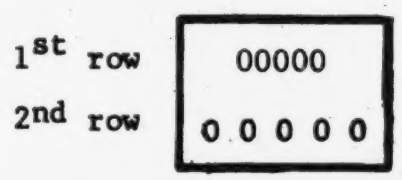
A. 0

B. 1

C. 4

D. 5

E. not necessarily any of these

13. 

A child says that there are more circles when they are spread out (as in the second row above) than when they are close together. This most probably means he is not yet able to

A. conserve numerousness

B. count from one to five

C. memorize simple addition facts

D. recognize when one stick is shorter than another

E. say "five" when presented with the numeral 5

14. If set A is equivalent to set B and set B is equivalent to set C, then

(I) A and C are equivalent

(II) A and C have the same members

(III) A and C can be put into one-to-one correspondence

A. I only

B. II only

C. I and III only

D. II and III only

E. I, II and III

15. Which number is associated with the empty set?

A. -1

B. 0

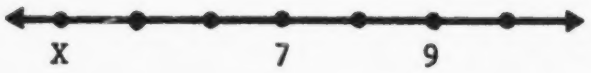
C. 1

D. 10

E. ∞

PIA-specific content: Grade 2

1.



On the number line above, X =

A. 0

B. 1

C. 4

D. 5

E. not necessarily any of these

2. When performing the indicated subtraction, 34 should be considered as

$$\begin{array}{r} 34 \\ -\ 18 \\ \hline 16 \end{array}$$

A. 20 tens, 4 ones

B. 20 tens, 14 ones

C. 30 tens, 4 ones

D. 2 tens, 14 ones

E. 3 tens, 14 ones

3. Which of the following is exact? Finding out the number of

A. pounds a boy weighs

B. inches in a foot

C. cups of water in a bowl

D. miles between two cities

E. square inches in the area of this page

4. Which is an example of changing something from a closed curve to an open curve?

A. breaking a rubber band

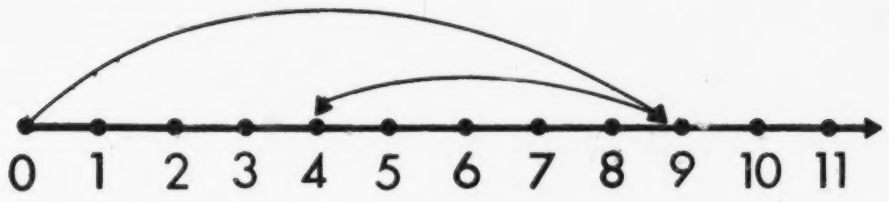
B. peeling a tangerine

C. bending a hula hoop

D. cutting a hole in a fish net

E. snapping a rung in a 6-foot ladder

5.



This number line diagram shows which of the following situations?

A. $4 + 5 = 9$

B. $9 - 4 = 5$

C. $9 - 5 = 4$

D. $9 - 4 - 5 = 0$

E. $0 + 4 + 5 = 9$

6. Which point is inside the curve?

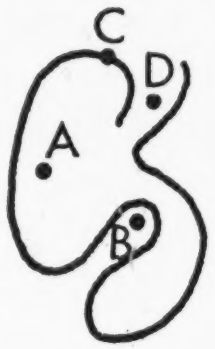
A. A

B. B

C. C

D. D

E. none of these



7. How many members are in the solution set for $n < 6$ if the solution set is drawn from the set of positive odd numbers?

A. 3

B. 4

C. 5

D. an infinite number

E. none of these

8. The shaded area of the polyhedron is called a

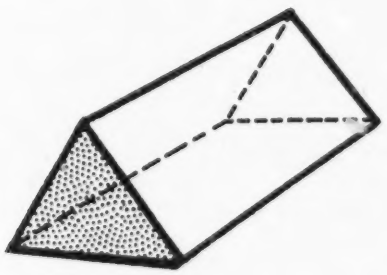
A. prism

B. pyramid

C. edge

D. face

E. vertex



9. Which of these would be solved using subtraction?

A. $n + 7 = 12$

B. $n - 7 = 12$

C. $12 + 7 = n$

D. $n - 12 = 7$

E. none of these

10. In order for a pupil to decide which is the larger of 248 and 263, it is sufficient for him to compare the

A. tens

B. hundreds

C. tens and ones

D. hundreds and ones

E. hundreds and tens

11. If a, b, c, d and e are whole numbers, and if $a > d$, $b < c$, $e > c$ and $a < b$, which is the largest number?

A. a

B. b

C. c

D. d

E. e

12. Which of these shows the meaning of inverse operations?

A. $3 - 3 = 0$

B. $3 + 0 = 3$; $3 - 0 = 3$

C. $3 + 5 = 5 + 3$

D. $(3 + 5) - 5 = 3$

E. $(3 + 5) + 0 = 3 + (5 + 0)$

13. 419 can be rewritten

A. 2 hundreds + 10 tens + 9 ones

B. 2 hundreds + 20 tens + 19 ones

C. 3 hundreds + 9 tens + 19 ones

D. 3 hundreds + 10 tens + 9 ones

E. 3 hundreds + 20 tens + 19 ones

GPO 827-794—6

14. Geometric figures which are the same size and shape are

A. equal

B. equivalent

C. proportional

D. symmetric

E. congruent

15. The column which is shaded in the table shows that

A. $n + 0 = n$

B. $n \times 1 = n$

C. $n \times 0 = 0$

D. $n \div n = 1$

E. $n + r = r + n$

	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	5
2	2	3	4	5	6
3	3	4	5	6	7
4	4	5	6	7	8

PIA-specific content: Grade 3

1.

The equality of the arrays above illustrates which of the following properties?

A. Commutative property of addition

B. Commutative property of multiplication

C. Associative property of addition

D. Associative property of multiplication

E. Distributive property of multiplication with respect to addition

2. Which of these number pairs (ratios) would <u>NOT</u> be used when converting between feet and yards?

A. $\frac{2}{9}$

B. $\frac{12}{4}$

C. $\frac{6}{18}$

D. $\frac{51}{17}$

E. $\frac{9}{27}$

3. Which point is inside the curve?

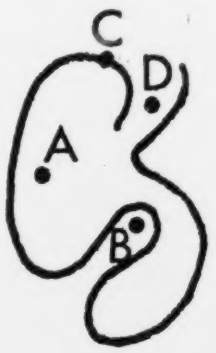
A. A

B. B

C. C

D. D

E. none of these



4. When performing the indicated subtraction, 34 should be considered as

$$\begin{array}{r} 34 \\ -18 \\ \hline 16 \end{array}$$

A. 20 tens, 4 ones

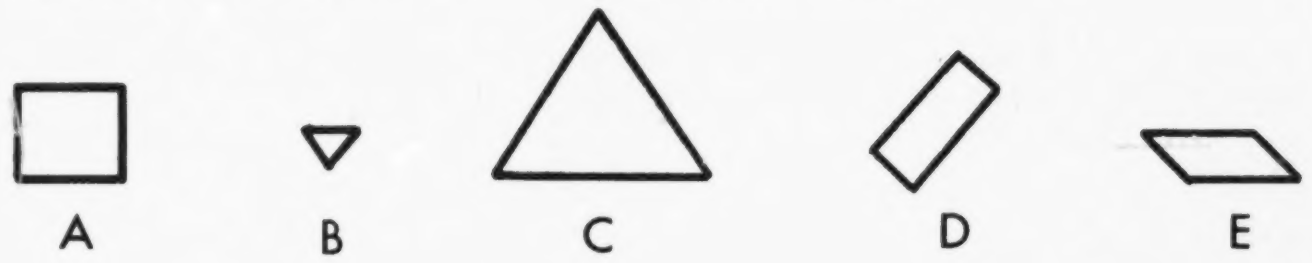
B. 20 tens, 14 ones

C. 30 tens, 4 ones

D. 2 tens, 14 ones

E. 3 tens, 14 ones

5. Which of these figures contains the largest angle?



6. What is the name of the number property that links addition with multiplication?

A. Associative
B. Commutative
C. Distributive
D. Identity
E. Inverse

7. Which of the following is exact? Finding out the number of

A. pounds a boy weighs
B. inches in a foot
C. cups of water in a bowl
D. miles between two cities
E. square inches in the area of this page

8. Geometric figures which are the same size and shape are

A. equal
B. equivalent
C. proportional
D. symmetric
E. congruent

9. Which of these shows the meaning of inverse operations?

A. $3 - 3 = 0$
B. $3 + 0 = 3;\ 3 - 0 = 3$
C. $3 + 5 = 5 + 3$
D. $(3 + 5) - 5 = 3$
E. $(3 + 5) + 0 = 3 + (5 + 0)$

10. If a, b, c, d and e are whole numbers, and if $a > d$, $b < c$, $e > c$ and $a < b$, which is the largest number?

A. a

B. b

C. c

D. d

E. e

11. Which of these is <u>NOT</u> a factor of 12?

A. 1

B. 2

C. 4

D. 9

E. 12

12. The basic property used in the algorism to the right is illustrated by which of the following?

$$\begin{array}{r} 432 \\ \times\ \ 2 \\ \hline 4 \\ 60 \\ 800 \\ \hline 864 \end{array}$$

A. a(bc) = (ab)c

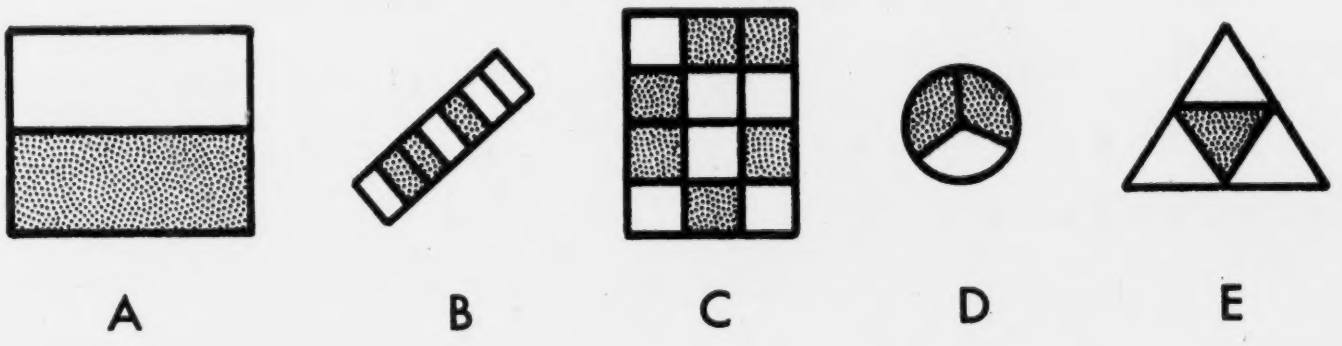
B. a + (b +c) = a + (c + b)

C. a(b + c) = ab + ac

D. a + b + c = c + b + a

E. bc = cba

13. Which shaded area represents the largest fraction?



A B C D E

14. The equality $2 \times (3+4) = 2 + (3 \times 4)$ is an illustration of which of the following principles?

A. The associative law of addition

B. The commutative law of addition

C. The distributive law of multiplication with respect to addition

D. The commutative law of multiplication

E. None of these

15. The column which is shaded in the table shows that

A. $n + 0 = n$

B. $n \times 1 = n$

C. $n \times 0 = 0$

D. $n \div n = 1$

E. $n + r = r + n$

	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	5
2	2	3	4	5	6
3	3	4	5	6	7
4	4	5	6	7	8

PIA-specific content: Grade 4

1. Which of these is NOT a factor of 12?

A. 1

B. 2

C. 4

D. 9

E. 12

2. Which number sentence would most appropriately be solved by division?

A. $n \times 8 = 24$

B. $3 \times 8 = n$

C. $8 \times 3 = n \times 3$

D. $n \times 8 = 8 \times n$

E. $n + 8 + 8 = 24$

3. Which statement about geometric figures is false?

A. a cube has 8 faces

B. two congruent triangles are similar

C. two perpendicular lines form four right angles

D. if the foci of an ellipse coincide, the ellipse is a circle

E. a triangle with sides 3, 4, and 5 is similar to one with sides 9, 12, and 15

4. The basic property used in the algorism to the right is illustrated by which of the following?

$$\begin{array}{r} 432 \\ \times\ \ 2 \\ \hline 4 \\ 60 \\ 800 \\ \hline 864 \end{array}$$

A. $a(bc) = (ab)c$

B. $a + (b + c) = a + (c + b)$

C. $a(b + c) = ab + ac$

D. $a + b + c = c + b + a$

E. $abc = cba$

5. Geometric figures which are the same size and shape are

A. equal

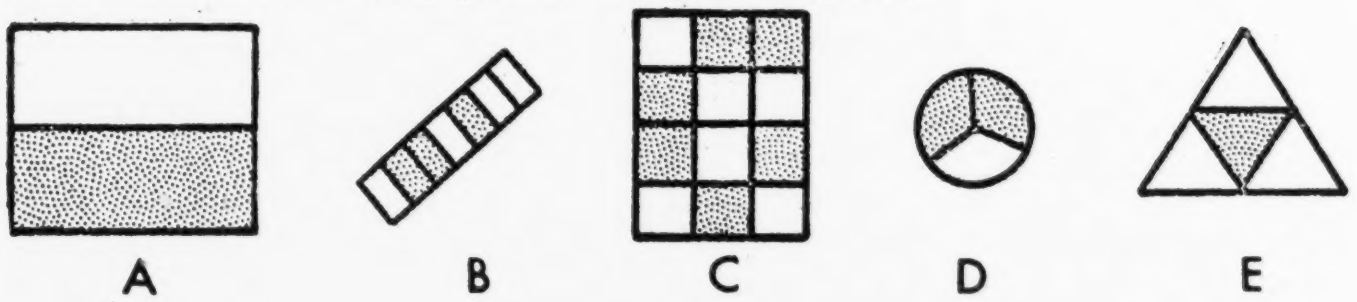
B. equivalent

C. proportional

D. symmetric

E. congruent

6. Which shaded area represents the largest fraction?



7. If a, b, c, d and e are whole numbers, and if $a > d$, $b < c$, $e > c$ and $a < b$, which is the largest number?

A. a

B. b

C. c

D. d

E. e

8. Which of the following is exact? Finding out the number of

A. pounds a boy weighs

B. inches in a foot

C. cups of water in a bowl

D. miles between two cities

E. square inches in the area of this page

9. The equality $2 \times (3+4) = 2 + (3 \times 4)$ is an illustration of which of the following principles?

A. The associative law of addition

B. The commutative law of addition

C. The distributive law of multiplication with respect to addition

D. The commutative law of multiplication

E. None of these

10. Which number sentence represents this story problem:

"Carlos has 40 pennies in his coin collection. He can put 8 pennies on each page of his coin book. How many pages can he fill?"

A. $n \times 8 = 40$

B. $8 + n = 40$

C. $8 \times 40 = n$

D. $8 \div n = 40$

E. $40 - 8 = n$

11.

The equality of the arrays above illustrates which of the following properties?

A. Commutative property of addition

B. Commutative property of multiplication

C. Associative property of addition

D. Associative property of multiplication

E. Distributive property of multiplication with respect to addition

12. Which of the following ratios is not equivalent to the others?

A. $\frac{4}{6}$

B. $\frac{6}{9}$

C. $\frac{9}{12}$

D. $\frac{12}{18}$

E. $\frac{18}{27}$

13. What number property explains why the multiplication table is symmetric about the dotted line?

A. $n \times 0 = 0$

B. $n \times 1 = n$

C. $n \div n = 1$

D. $n \times r = r \times n$

E. none of these

x	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	6	8
3	0	3	6	9	12
4	0	4	8	12	16

14. Which of the following pairs of sentences are equivalent?

(I)	$8 + n = 13$	and	$13 - 8 = n$
(II)	$7 + n = 10$	and	$10 - n = 7$
(III)	$n + 3 = 9$	and	$19 - 13 = n$

A. only I

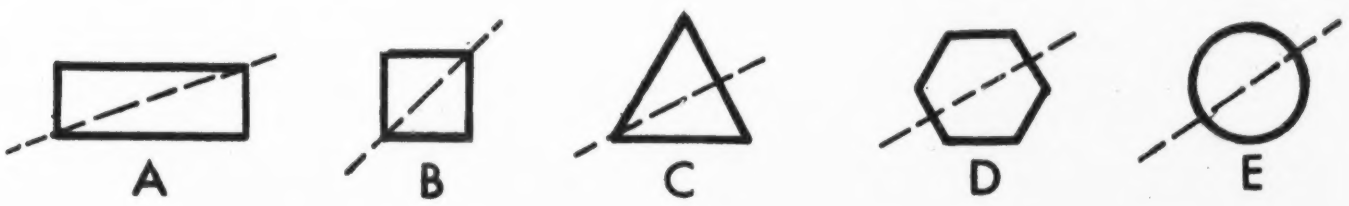
B. only I and II

C. only I and III

D. only II and III

E. I, II and III

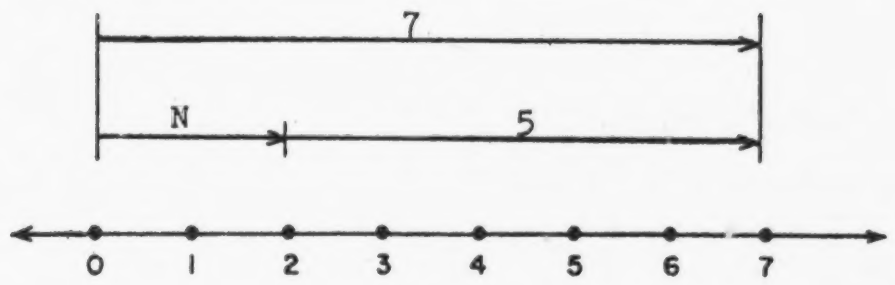
15. Which figure below does <u>NOT</u> show a line of symmetry?



Basic Mathematics

Questions 16-35

16. Which of the following number sentences are equivalent to the sentence whose model is shown on this number line?



(a) N = 7 + 5

(b) N = 7 - 5

(c) N + 5 = 7

(d) 7 - N = 5

A. Only (a) and (b)

B. Only (b)

C. Only (b) and (c)

D. Only (b), (c) and (d)

E. All of these

17. If x and y are positive even integers and if p and q are positive odd integers, which of the following is FALSE?

A. x + y is a positive even integer

B. p + q is a positive odd integer

C. xy is a positive even integer

D. pq is a positive odd integer

E. px is a positive even integer

Questions 18 and 19 refer to the addition exercise below:

$$
\begin{array}{rl}
 & 672 = 6 \text{ hundreds} + 7 \text{ tens} + 2 \text{ ones} \\
+ & \underline{834 = 8 \text{ hundreds} + P \text{ tens} + 4 \text{ ones}} \\
 & Q \text{ hundreds} + 10 \text{ tens} + 6 \text{ ones} \\
 & = R \text{ thousands} + S \text{ hundreds} + T \text{ tens} + U \text{ ones} \\
 & = R, S\ T\ U \text{ (Final answer)}
\end{array}
$$

18. P =

A. 8

B. 3

C. 83

D. 34

E. 834

19. In the final answer, R, S T U represents:

A. The product of the numbers R, S, T and U

B. The sum of the numbers R, S, T and U

C. (R x 1000) + (S x 100) + (T x 10) + (U x 1)

D. (R x 1) + (S x 10) + (T x 100) + (U x 1000)

E. Eleven thousand, five hundred six

20. In order to find the product 6 x 9 we might:

A. Form the union of two disjoint sets with six and nine members

B. Form a six by nine array

C. Use the division algorithm

D. Notice that $(6 \times 9)_{\text{base 10}} = 3_{\text{ten}} \times (6_{\text{three}} \times 9_{\text{three}})$.

E. Draw a triangle with sides 6" and 9" long.

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21.

$$\begin{array}{r} 5 \\ \times\ 4 \\ \hline 20 \end{array}$$

Which of the following checks of the computation above makes direct use of the commutative property?

A. $4\overline{)20}$ $\quad\begin{array}{r}5\\ 20\\ \hline 0\end{array}$

B. $5\overline{)20}$ $\quad\begin{array}{r}4\\ 20\\ \hline 0\end{array}$

C. $\begin{array}{r}5\\5\\5\\5\\ \hline 20\end{array}$

D. $\begin{array}{r}4\\4\\4\\4\\ \hline 20\end{array}$

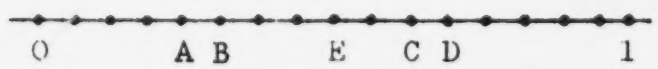
E. $\begin{array}{r}4\\ \times\ 5\\ \hline 20\end{array}$

22. $4 \times 376 = (4 \times 300) + \square + (4 \times 6)$

What number goes in place of $\square$ above?

A. 12
B. 24
C. 28
D. 240
E. 280

23. Which of the points A, B, C, D or E on this number line can be named by the fraction $\frac{5}{8}$?



24. Choose the best way to complete this statement: Standard units of measurement are used because

A. it is important for people to use the same unit in dealing with each other.

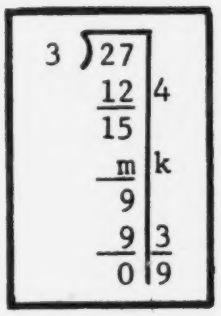
B. standard units give more accurate measurements than units which are not standard.

C. people have always used them.

D. they are all related to base 10 numeration.

E. the English system is more natural than the metric system.

25.



If the algorism above is based on interpreting division as successive subtraction, then k =

A. 2

B. 3

C. 5

D. 6

E. 9

26. Suppose $\_\_|\_\_$ is a binary operation with

$1 \_\_|\_\_ 1 = 0$ $\quad$ $7 \_\_|\_\_ 2 = 13$

$2 \_\_|\_\_ 2 = 3$ $\quad$ $4 \_\_|\_\_ 4 = 15$

$5 \_\_|\_\_ 6 = 29$ $\quad$ $9 \_\_|\_\_ 2 = 17$

What is $6 \_\_|\_\_ 3$?

A. 6

B. 3

C. 9

D. 17

E. 18

27. The inverse operation for addition is:

A. addition

B. subtraction

C. multiplication

D. division

E. none of these

28. A point P lies 4.03 inches from the center of a circle with a radius of 4.3 inches. Which of the following is true?

A. P lies outside the circle

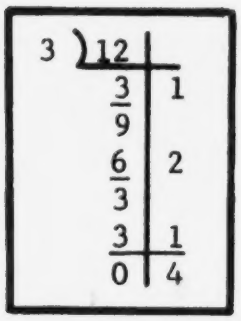
B. P lies at the center

C. P lies inside the circle, but not at the center

D. P lies somewhere on the circumference

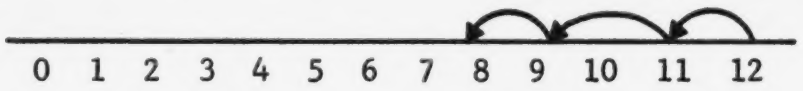
E. It cannot be determined whether P lies inside, on, or outside the circle

29.

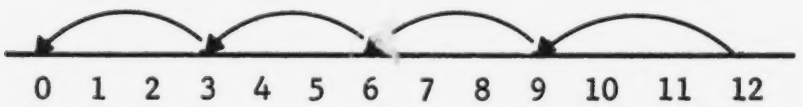


Which of the following number-line diagrams illustrates the subtraction operations depicted in the division alorism above?

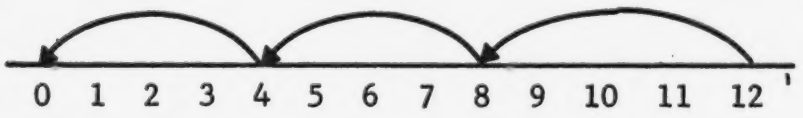
A.



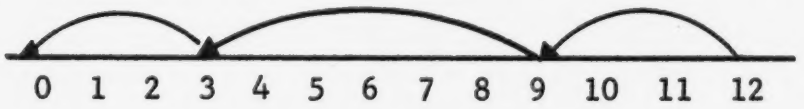
B.



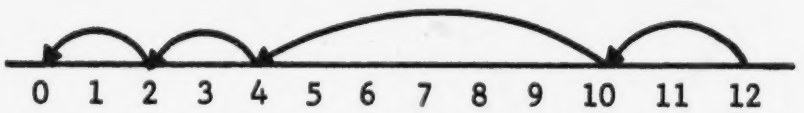
C.



D.

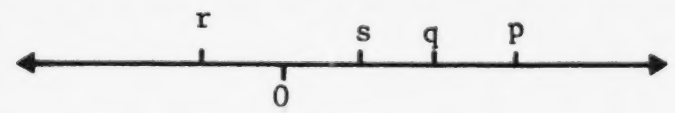


E.

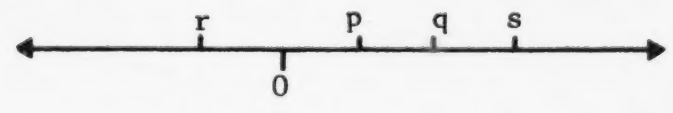


30. If $r < 0$, $s > p$, $s < q$, and $0 < p$, which of the following number lines shows the correct relative positions of p, q, r, and s?

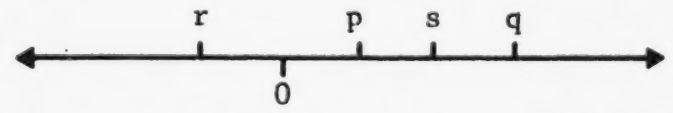
A.



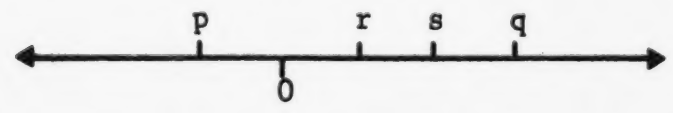
B.



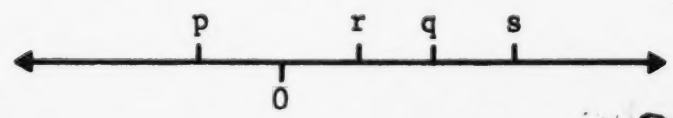
C.



D.



E.



31. If x, y, z are numerals in <u>base five</u> notation, which of the following are correct?

(a) $x + y = y + x$

(b) $x\,(y + z) = xy + xz$

(c) if $x < y$ and $y < z$ then $x < z$

A. Only (a)

B. Only (b)

C. Only (a) and (b)

D. Only (a) and (c)

E. (a), (b), and (c)

32. If A and B are two sets which can be matched (put in one-to-one correspondence then:

A. A and B must both be sets of numbers

B. A and B must have the same number of elements

C. Neither A nor B can be the empty set

D. A and B must both be sets of points

E. A and B must each have one member

33. Which of the following is the multiplicative inverse of 4.0?

A. 0.04

B. 0.25

C. 0.4

D. 0.5

E. 1.25

34. Which one of the following division examples will result in the quotient 25.2?

A. $5\overline{)\ .126}$

B. $.5\overline{)\ .126}$

C. $.05\overline{)\ .126}$

D. $.005\overline{)\ .126}$

E. $.0005\overline{)\ .126}$

35. A football team starts the following series of plays from its own 20-yard line: gains 8 yards, loses 2 yards, loses 1 yard, gains 6 yards. On what yard line is the ball now?

A. 11

B. 26

C. 31

D. 34

E. 37

When you have finished, please check to make sure you have answered <u>all</u> questions on the questionnaire and on this test. Then hand in this booklet along with your honorarium form.

Thank you for taking part in this project.

Appendix F
Additional Statistical Tables and Figure

Table 1-F
Sample Mean Scores by Treatment by Site (Obtained from Averages over Grade)

Variable	SITE														
	Urban Site 1			Urban Site 2			Rural Site 1			Rural Site 2			Average[a]		
	Sample PIA	Control	Ave.	Sample PIA	Control	Ave.	Sample PIA	Control	Ave.	Sample PIA	Control	Ave.	All Sites PIA	Control	Ave.
Knowledge: Pretests															
Basic Math[b]	12.93	11.36	12.15	10.30	9.36	9.83	10.63	10.09	10.36	13.66	10.28	11.97	11.88	10.27	11.08
Other Pretest Items[c]	11.57	10.93	11.25	10.02	9.13	9.57	9.75	9.58	9.66	11.48	8.76	10.12	10.70	9.60	10.15
Total	24.50	22.29	23.40	20.32	18.49	19.40	20.38	19.67	20.02	25.14	19.04	22.09	22.59	19.87	21.23
Knowledge: Posttest															
PIA-Specific[d]	9.52	8.34	8.93	8.90	7.59	8.24	7.88	8.80	8.34	9.75	7.20	8.48	9.01	7.92	8.47
Basic Math[b]	14.02	13.67	13.84	9.70	10.79	10.24	11.72	12.40	12.06	15.10	12.48	13.79	12.63	12.33	12.48
Gain: Basic Math	1.09	2.31	1.70	-.60	1.43	.41	1.09	2.31	1.70	1.44	2.20	1.82	.75	2.06	1.41
Attitude: Pre-questionnaire															
Enjoyment[e]	8.93	8.57	8.75	7.84	7.26	7.55	8.14	7.72	7.93	7.94	9.18	8.56	8.21	8.18	8.20
Confidence[e]	6.11	6.86	6.48	7.00	6.15	6.58	6.28	6.07	6.17	5.85	6.60	6.23	6.31	6.42	6.36
Attitude: Post-questionnaire															
Enjoyment[e]	8.90	8.96	8.93	7.75	7.15	7.45	7.80	8.79	8.30	8.46	9.33	8.89	8.23	8.56	8.39
Confidence[e]	8.45	8.02	8.24	7.60	7.00	7.30	7.96	8.79	8.38	7.50	6.95	7.23	7.88	7.69	7.78
Sample Size - Schools	20	18	38	5	4	9	5	5	10	8	8	16	38	35	73
Teachers	69	78	147	54	38	92	35	36	71	21	18	39	179	170	349

[a]All averages are unweighted arithmetic means.

[b]20 items.

[c]30 items.

[d]15 items.

[e]For purposes of analysis and comparability, the enjoyment and confidence variables from the questionnaires were transformed so that each had a range from 1 = low to 13 = high.

Table 2-F
Weighted Mean Univariate Analysis of Variance, with Grade as Repeated Measure Within School

Source	df for Hypothesis	Results: PIA-Specific MS	PIA-Specific F	PIA-Specific p	Basic Mathematics MS	Basic Mathematics F	Basic Mathematics p	Direction If Relevant
Mean	1	10473.98	---	---	23444.22	---	---	
Among Schools[a]	72							
Among Cells	7							
Among Sites	3	4.464	0.99	.40	78.654	9.55	.0001*	
Treatment[b]	1	29.975	6.66	.013*	1.758	0.21	.65	PIA higher
Treatment x Site	3	11.742	2.61	.06	12.034	1.46	.23	
Within Cells	65	6.040	1.34	.13	13.629	1.65	.03*	
Within Schools	66							
Among Grades Specific to Sites	4	7.007	1.56	.20	2.525	0.31	.87	
Treatment x Grades Specific to Sites	4	2.452	0.54	.70	7.968	0.97	.43	
Schools x Grades Within Cells	58	4.501	---	---	8.240	---	---	

*Statistically significant with .05 level criterion.

[a]Among School analysis conforms to classical Among Units analysis for "repeated measures" analysis.

[b]Removed after site effects removed; assumes Treatment x Site effects are zero.

Table 3-F
Estimates of Parameters: Knowledge
Variations in Covariance Model
(Site x Treatment Effect Assumed Zero)

	Variable					
	PIA-Specific		Basic Math (post)		Gain: Basic Math	
Parameter	Estimate	S.E.[a] (Est.)	Estimate	S.E.[a] (Est.)	Estimate	S.E.[b] (Est.)
Covariates (Regression Coefficients)						
Basic Math Pretest (Average)	.380	.065	.807	.072	-.186	.071
Basic Math Pretest x Treatment	-.112	.065	.050	.072	---	---
Basic Math						
Specific to PIA	.268	.046	.857	.050	---	---
Without PIA	.492	.122	.756	.135	---	---
Target Population Mean	8.27[c]	.29	12.94[d]	.33	1.84	.30
Site Contrasts						
Urban-Rural	.42	.79	-1.49	.88	-1.53	.87
Urban Site 1-Urban Site 2	-.16	.56	1.76	.62	1.77	.61
Rural Site 1-Rural Site 2	.32	.59	-.36	.66	-.42	.65
Gross PIA Increment[e]	3.128	1.525	-2.039	1.693	-.89	.39
Average PIA Increment[e]	.658	.373	-.931	.415	-.89	.39

[a] $\underline{df} = 63$.

[b] $\underline{df} = 64$.

[c] Out of 15 items.

[d] Out of 20 items.

[e] The "gross" increment is the difference if the Basic Math Pretest Score is zero; the "average" increment is the increase at the mean Pretest Score of 11.1; e.g., the gross does not allow for variation in PIA effect by Pretest Score.

Table 4-F
Estimates of Parameters: Attitude
Variations in Covariance Model
(Site x Treatment Effect Assumed Zero)

	Variable			
	Enjoyment of Teaching Math		Confidence in Teaching Math	
Parameter	Estimate	Standard Error (Estimated)	Estimate	Standard Error (Estimated)
Covariates (Regression Coefficients)				
Enjoyment Pretest	.586	.131	.222	.119
Confidence Pretest	.120	.108	.405	.098
Target Population Mean[a]	8.535	.291	7.554	.269
Site Contrasts				
Urban-Rural	-.754	.884	-.301	.801
Urban Site 1-Urban Site 2	.766	.617	.731	.559
Rural Site 1-Rural Site 2	-.225	.652	1.309	.590
PIA Increment	-.260	.379	.464	.344

[a]For purposes of analysis, the enjoyment and confidence variables from the questionnaire were transformed so that each had a possible range from 1 = low to 13 = high.

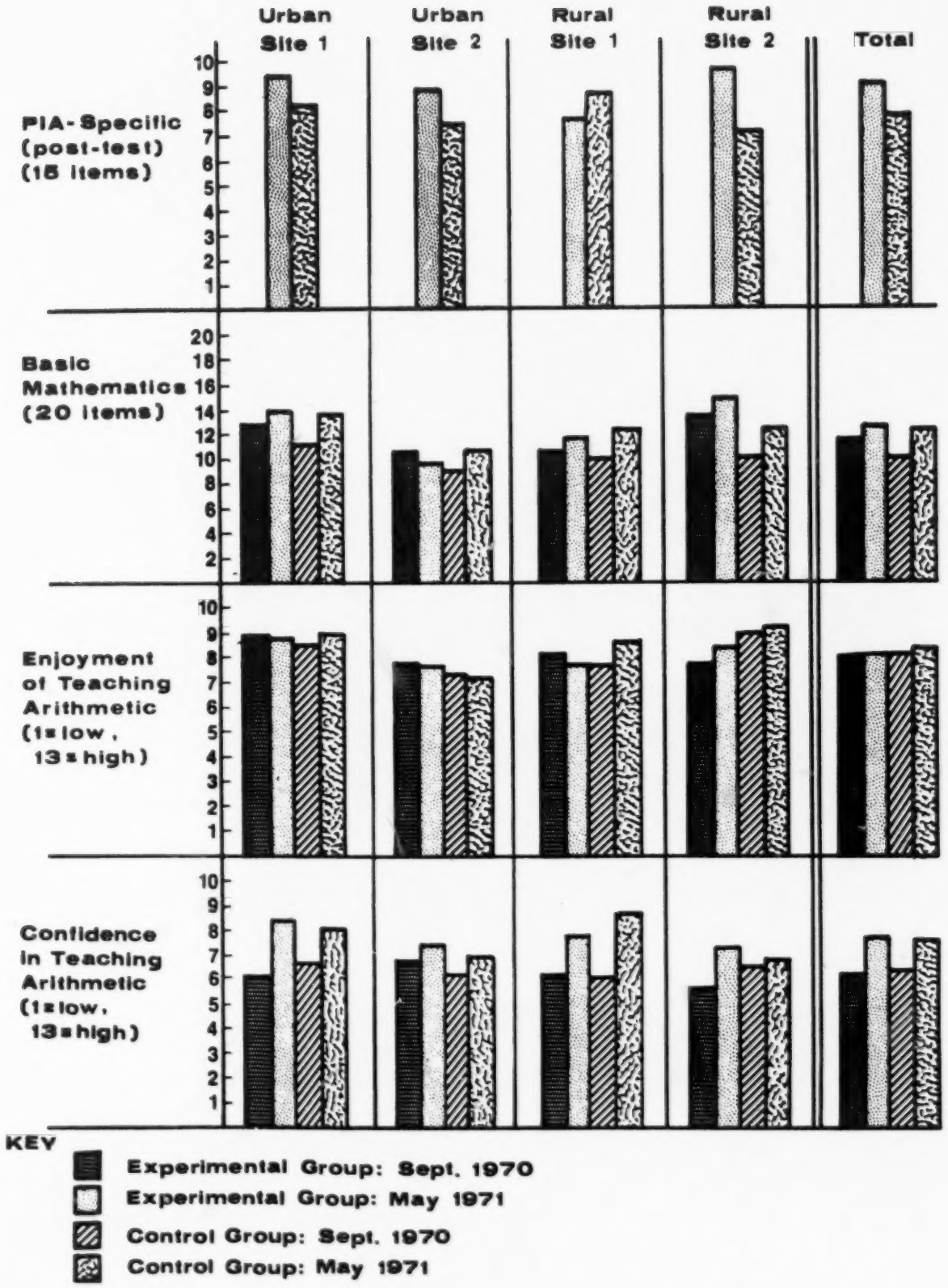


Fig. 1-F. Performance levels on knowledge and attitude variables for experimental and control teachers: Sample data.

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